MHT CET FULL TEST-1
ANSWER KEY

MATHEMATICS

Q.1 (D)	**Q.2** (C)	**Q.3** (B)	**Q.4** (D)	**Q.5** (A)	**Q.6** (D)	**Q.7** (A)	**Q.8** (B)	**Q.9** (A)	**Q.10** (C)
Q.11 (B)	**Q.12** (B)	**Q.13** (C)	**Q.14** (D)	**Q.15** (C)	**Q.16** (A)	**Q.17** (C)	**Q.18** (B)	**Q.19** (D)	**Q.20** (C)
Q.21 (C)	**Q.22** (A)	**Q.23** (A)	**Q.24** (D)	**Q.25** (B)	**Q.26** (A)	**Q.27** (B)	**Q.28** (C)	**Q.29** (D)	**Q.30** (D)
Q.31 (A)	**Q.32** (B)	**Q.33** (D)	**Q.34** (B)	**Q.35** (A)	**Q.36** (C)	**Q.37** (A)	**Q.38** (A)	**Q.39** (C)	**Q.40** (B)
Q.41 (B)	**Q.42** (D)	**Q.43** (B)	**Q.44** (C)	**Q.45** (A)	**Q.46** (B)	**Q.47** (B)	**Q.48** (A)	**Q.49** (D)	**Q.50** (B)

SOLUTIONS

MATHEMATICS

Q.1 (D)

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

now $\Delta = a^2 + b^2 - c^2$

hence $\cos C = \dfrac{\Delta}{2ab}$ ....(1)

also $\Delta = \dfrac{1}{2} ab \sin C \Rightarrow \dfrac{2\Delta}{\sin C} = ab$

$\Rightarrow \sin C = \dfrac{2\Delta}{ab}$ ....(2)

from (1) and (2)

$\tan C = \dfrac{2\Delta}{ab} \cdot \dfrac{2ab}{\Delta} = 4$. **Ans.**

Q.2 (C)

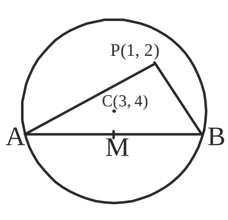
slope = – 3/5

equation of the line is $3x + 5y = 43$

$5y = 43 - 3x \Rightarrow y = \dfrac{43 - 3x}{5}$

Hence point are (1, 8), (6, 5) , (11, 2)

Q.3 (B)



$PM^2 = BM^2 = r^2 - CM^2$

$\Rightarrow (h-1)^2 + (k-2)^2 + (h-3)^2 + (k-4)^2 = 36$

$\Rightarrow x^2 + y^2 - 4x - 6y - 3 = 0$

So $a = 2, b = 3, c = 3$

$a + b + c = 8$

Q.4 (D)

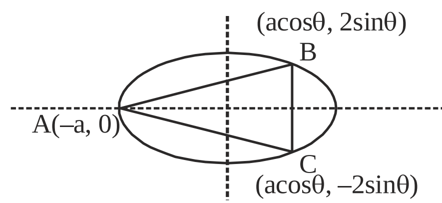
$2\sin x + 1 - 2\sin^2 x = 1$

$\Rightarrow 2\sin x\,(1 - \sin x) = 0$

$\Rightarrow \sin x = 0$ or $\sin x = 1$

$\Rightarrow x = 0, \dfrac{\pi}{2}, \pi, 2\pi.$

Q.5 (A)



$\text{area} = \dfrac{1}{2}(4\sin\theta)(a + a\cos\theta)$

$\dfrac{dA}{d\theta} = 2a(-\sin^2\theta + (1+\cos\theta)\cos\theta) = 0$

maximum when $\cos\theta = \dfrac{1}{2}$

$\Rightarrow A_{max} = 2a\dfrac{\sqrt{3}}{2}\left(1 + \dfrac{1}{2}\right) = 6\sqrt{3}$

$\Rightarrow a = 4$

$\Rightarrow e = \sqrt{1 - \dfrac{4}{16}} = \dfrac{\sqrt{3}}{2}$

Q.6 (D)

Given, $V = \pi r^2 h$

Differentiating both sides

$\dfrac{dV}{dt} = \pi\left(r^2\dfrac{dh}{dt} + 2r\dfrac{dr}{dt}h\right) = \pi r\left(r\dfrac{dh}{dt} + 2h\dfrac{dr}{dt}\right)$

$\dfrac{dr}{dt} = \dfrac{1}{10}$ and $\dfrac{dh}{dt} = -\dfrac{2}{10}$

$$\frac{dV}{dt} = \pi r\left(r\left(-\frac{2}{10}\right)+2h\left(\frac{1}{10}\right)\right) = \frac{\pi r}{5}(-r+h)$$

Thus, when r = 2 and h = 3,

$$\frac{dV}{dt} = \frac{\pi(2)}{5}(-2+3) = \frac{2\pi}{5}$$

Q.7 (A)

Let A : The coin is fair

B : The coin is biased

Now, $P(H) = P(A)\,P\left(\frac{H}{A}\right) + P(B)\,P\left(\frac{H}{B}\right)$ (using total law of probability)

$$\Rightarrow \left(\frac{n+1}{2n+1}\right)\times\frac{1}{2}+\left(\frac{n}{2n+1}\right)\times 1 = \frac{31}{42} \text{ (Given)}$$

$\Rightarrow n = 10$ **Ans.**

Q.8 (B)

Let $\theta = \sin^{-1}\frac{2}{3}$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta = 3.\frac{2}{3} - 4.\frac{8}{27}$$

$$= 2 - \frac{32}{27} = \frac{22}{27}$$

Q.9 (A)

Equation of the plane through – 1, 3, 2, is

$a(x+1) + b(y-3) + c(z-2) = 0$ ......(1)

(1) is perpendicular to $3 + 2y + 3z = 5$ and $3x + 3y + z = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{vmatrix} = \hat{i}(2-9) - \hat{j}(1-9) + \hat{k}(3-6)$$

vector perpedicular to two planes is $-7\hat{i} + 8\hat{j} - 3\hat{k}$.

Hence equation of plane is

$7(x+1) - 8(y-3) + 3(z-2) = 0$

$7x - 8y + 3z + 25 = 0 \equiv ax + by + cz + 25$

Hence + b + c = 7 – 8 +3 = 2 **Ans.**

Q.10 (C)

Let $z = a + bi$.

$|z|^2 = a^2 + b^2$.

$\because z + |z| = 2 + 8i$

$\therefore a + bi + \sqrt{a^2+b^2} = 2 + 8i$

$a + \sqrt{a^2+b^2} = 2, b = 8$

$a + \sqrt{a^2+64} = 2$

$a^2 + 64 = (2-a)^2 = a^2 - 4a + 4,$

$4a = -60, a = -15$. Thus, $a^2 + b^2 = 225 + 64 = 289$

$\therefore |z| = \sqrt{a^2+b^2} = \sqrt{289} = 17$ **Ans.**

Q.11 (B)

$$f(x) = \begin{cases} -x^2, & x<0 \\ x^2, & x>0 \end{cases} \Rightarrow f'(x) = \begin{cases} -2x, & x<0 \\ 2x, & x>0 \end{cases}$$

$\therefore f'(x) = 2|x|$.

Q.12 (B)

$$f(x) = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right] = 2\tan^{-1}(\log x)$$

$$f'(x) = 2.\frac{1}{1+(\log x)^2}.\frac{1}{x}.\text{Therefore } f'(e) = \frac{1}{e}$$

Q.13 (C)

Given numbers can be rearranged as

1 4 7 88	→	30
2 5 8 89	→	30
3 6 9 90	→	30

That means we must take two numbers from last row or one number each from first and second row.

Total ways = ${}^{30}C_2 + {}^{30}C_1 \cdot {}^{30}C_1 = 435 + 900$

= 1335 **Ans.**

Q.14 (D)

$$\vec{a}+\vec{b} = -\vec{c} \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = |\vec{c}|^2$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Q.15 **(C)**

$\because f(x) = \cos^4 x + 3(1-\cos^2 x)$

$= \cos^4 x - 3\cos^2 x + 3$

$$= \left(\cos^2 x - \frac{3}{2}\right)^2 + \frac{3}{4}$$

$$\because 0 \le \cos^2 x \le 1 \Rightarrow \frac{-3}{2} \le \cos^2 x - \frac{3}{2} \le \frac{-1}{2}$$

$$\Rightarrow \frac{1}{4} \le \left(\cos^2 x - \frac{3}{2}\right)^2 \le \frac{9}{4}$$

$$\Rightarrow 1 \le \left(\cos^2 x - \frac{3}{2}\right)^2 + \frac{3}{4} \le 3$$

$\therefore$ Number of integers in the range = 3

Q.16 (A)

$2\sin^2 2\theta = b^{5/4}$ ......(A)

also $5\sin^2\theta = 2 + \cos^2\theta = 3 - \sin^2\theta$

$6\sin^2\theta = 3 \Rightarrow \sin^2\theta = \frac{1}{2} \therefore \cos^2\theta = \frac{1}{2}$

$8\sin^2\theta \cos^2\theta = b^{5/4}$

$\Rightarrow 8 \cdot \frac{1}{2}\left(\frac{1}{2}\right) = b^{5/4} \Rightarrow 2 = b^{5/4}$

$\therefore b = (B)^{4/5} \Rightarrow q = \frac{4}{5}$.

Q.17 (C)

$$\lim_{x\to 0} \frac{\sin^4(3\sqrt{x})}{1-\cos x}$$

$$\lim_{x\to 0} \frac{\sin^4(3\sqrt{x})}{\left(3\sqrt{x}\right)^4}\frac{x^2}{(1-\cos x)}\times 3^4$$

Q.18 (B)

For continuous function

$f(0^-) = f(0) = f(0^+)$

$f(0^-) = (P+1)+1 = P+2$

$f(0) = q$

$$f(0^+) = \lim_{x\to 0^+}\frac{\sqrt{1+x}-1}{x} = \lim_{x\to 0^+}\frac{1}{\sqrt{1-x}+1} = \frac{1}{2}$$

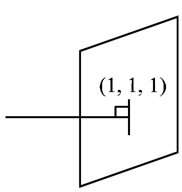
$q = \frac{1}{2}$; $P+2 = q \quad \Rightarrow P = \frac{-3}{2}$

Q.19 (D)

Negation of $(P \wedge \sim R) \leftrightarrow Q$ is $\sim((P \wedge \sim R) \leftrightarrow Q)$

It may also be written as $(\sim P \vee R) \leftrightarrow Q$

Q.20 (C)



Equation of the plane

$A(x-1) + B(y-1) + C(z-1) = 0 \quad ....(1)$

Since the line is perpendicular to the plane (1)

$\therefore 3(x-1) + 0(y-1) + 4(z-1) = 0$

$3x + 0y + 4z - 7 = 0$

distance from (0, 0, 0)

$$d = \left|\frac{-7}{5}\right| = \frac{7}{5} \Rightarrow (C)$$

Q.21 (C)

$$\begin{bmatrix}2 & -3\\4 & 0\end{bmatrix} - \begin{bmatrix}a & c\\b & d\end{bmatrix} = \begin{bmatrix}1 & 4\\2 & -5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}a & c\\b & d\end{bmatrix} = \begin{bmatrix}2 & -3\\4 & 0\end{bmatrix} - \begin{bmatrix}1 & 4\\2 & -5\end{bmatrix} = \begin{bmatrix}1 & -7\\2 & 5\end{bmatrix}$$

$\Rightarrow (a, b, c, d) = (1, 2, -7, 5)$.

Q.22 (A)

$\sin x - \left(1-\sin^2 x\right) + \frac{5}{4} = 0$

$\sin^2 x + \sin x + \frac{1}{4} = 0$

$\left(\sin x + \frac{1}{2}\right)^2 = 0 \Rightarrow \sin x = -\frac{1}{2}$

$\Rightarrow x = n\pi + (-1)^n\left(\frac{-\pi}{6}\right)$

Q.23 (A)

$[\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}] = m[\vec{a}\;\vec{b}\;\vec{c}]$

$\Rightarrow 2[\vec{a}\;\vec{b}\;\vec{c}] = m[\vec{a}\;\vec{b}\;\vec{c}] \quad m = 2$

Q.24 (D)

$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z-k}{3} = \lambda$

$\Rightarrow (\lambda-2, 2\lambda+3, 3\lambda+k)$ for A, $\lambda = 2$

$A(0, 7, 6+k)$

$\Rightarrow$ for B $\lambda = -\frac{k}{3}$

$\Rightarrow B\left(-2-\frac{k}{3}, 3-\frac{2k}{3}, 0\right)$

$\angle AOB = 90°$

$\Rightarrow \overrightarrow{AO} \cdot \overrightarrow{OB} = 0$

$\Rightarrow 7\left(-3+\frac{2k}{3}\right) = 0$

or $k = \frac{9}{2} \Rightarrow 2k = 9$

Q.25 (B)

$\int \sin 5x \cos 3x dx = \frac{1}{2}\int(\sin 8x + \sin 2x)dx$

$= \frac{-\cos 8x}{16} - \frac{\cos 2x}{4} + c$

Equating to the given value, we get $A = \frac{-\cos 2x}{4} + c$

Q.26 (A)

$f(x) = x^4 - \frac{x^3}{3} \Rightarrow f'(x) = 4x^3 - x^2$

For increasing $4x^3 - x^2 > 0 = x^2(4x - 1) > 0$

Therefore, the function is increasing for $x > \frac{1}{4}$

Similarly decreasing for $x < \frac{1}{4}$.

Q.27 (B)

We have, $2 \cos x (\operatorname{cosec} x - 2) - (\operatorname{cosec} x - 2) = 0$

$\Rightarrow (2 \cos x - 1)(\operatorname{cosec} x - 2) = 0$

$\therefore \cos x = \frac{1}{2}$ or $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}$.

Q.28 (C)

Let $A_n = \int_0^1 x^3(1-x^2)^n \, dx$

Put $x^2 = t \Rightarrow x\,dx = \frac{1}{2}\,dt$

$\therefore A_n = \frac{1}{2}\int_0^1 t\,(1-t)^n dt$

$= \frac{1}{2}\int_0^1 t^n\,(1-t)\,dt$ (Using King property)

$\Rightarrow A_n = \frac{1}{2}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$

Q.29 (D)

Let slopes are $m_1 = m$ & $m_2 = 4m$

$m_1 + m_2 = \frac{-2h}{b} \Rightarrow 5m = -10$

$m_1 m_2 = \frac{a}{b} \Rightarrow 4m^2 = a \Rightarrow a = 16$

Q.30 (D)

$\vec{p} \times (\vec{p} \times \vec{q})$

$= (\vec{p}\cdot\vec{q})\vec{p} - (\vec{p}\cdot\vec{p})\vec{q} = -4\vec{q}$

$\therefore \quad \vec{V} = -4\vec{p} \times (\vec{p} \times \vec{q})$

$= -4[(\vec{p}\cdot\vec{q})\,\vec{p} - (\vec{p}\cdot\vec{p})\vec{q}]$

$= +(4)(4)\,\vec{q} = 16\vec{q}$ **Ans.**

Q.31 (A)

Put $x = \sin\theta$, we get $\frac{d}{dx}\sin^{-1}(3x - 4x^3)$

$= \frac{d}{dx}\sin^{-1}(\sin 3\theta) = \frac{3}{\sqrt{1-x^2}}$

Q.32 (B)

$$\left.\begin{aligned} |3\vec{a} + 4\vec{b}|^2 &= 9\lambda^2 + 16\lambda^2 = 25\lambda^2 \\ |4\vec{a} + 3\vec{b}|^2 &= 16\lambda^2 + 9\lambda^2 = 25\lambda^2 \end{aligned}\right]$$

[Let $\lambda = |\vec{a}| = |\vec{b}|$ (say)]

given, $5\lambda + 5\lambda = 20 \Rightarrow \lambda = 2 \Rightarrow |\vec{a}| = |\vec{b}| = 2$

$\Rightarrow |(\vec{a} \times \vec{b}) \times \vec{a}| = |\vec{a} \times \vec{b}|\,|\vec{a}| \sin 90^\circ = |\vec{a}|^2 \times |\vec{b}| = 8$

Q.33 (D)

Given $y = \sqrt{9 - 2x^2}$

Put $y = x$, we get

$x = \sqrt{9 - 2x^2} \Rightarrow x^2 = 9 - 2x^2$ (squaring on both sides)

$\Rightarrow \quad 3x^2 - 9 \Rightarrow x^2 = 3$

$\therefore \quad x = \pm\sqrt{3}$

So, $(x = \sqrt{3},\ y = \sqrt{3})$ and

$(x = -\sqrt{3},\ y = -\sqrt{3})$ (reject)

Now, $\left.\frac{dy}{dx}\right]_{(\sqrt{3}, \sqrt{3})} = \frac{(-4x)}{2\sqrt{9-2x^2}} = \frac{-2x}{\sqrt{9-2x^2}}$

$= \frac{-2\sqrt{3}}{\sqrt{3}} = -2$

So, equation of tangent is $(y - \sqrt{3}) = -2(x - \sqrt{3})$

$\Rightarrow 2x + y = 3\sqrt{3}$ **Ans.**

Q.34 (B)

We have

$L_1 = \frac{x+3}{1} = \frac{y-1}{-2} = \frac{z+2}{1} = \lambda$ (let) and

$L_2 = \frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3} = \mu$ (say)

The shortest distance between L_1 ande L_2 is given by

$= \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$

$$= \left| \frac{(4\hat{i}+\hat{j}+5\hat{k})\cdot(-5\hat{i}-\hat{j}+3\hat{k})}{\sqrt{25+1+9}} \right|$$

$$= \left| \frac{-20-1+15}{\sqrt{35}} \right|$$ **Ans.**

Q.35 (A)

$$\int \frac{\text{cosec}\theta - \cot\theta}{\text{cosec}\theta + \cot\theta} d\theta = \int (\text{cosec}\theta - \cot\theta)^2 d\theta$$

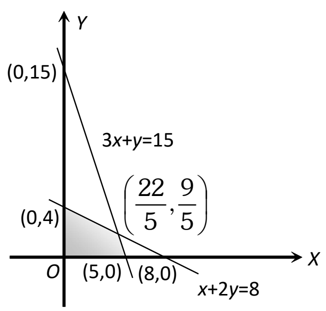
$$= \int \text{cosec}^2\theta \;\; d\theta + \int \cot^2\theta \;\; d\theta - 2\int \text{cosec}\theta \cot\theta \;\; d\theta$$

$$= \int (2\text{cosec}^2\theta - 1) \;\; d\theta - 2\int \text{cosec}\theta\cot\theta \;\; d\theta$$

$2 \text{ cosec } \theta - 2 \cot \theta - \theta + c$

Q.36 (C)

Max $z = 10x + 5y$ is at $\left(\frac{22}{5}, \frac{9}{5}\right)$ i.e. 53.



Q.37 (A)

$(\sim T \vee F) \wedge \sim T \Rightarrow T$

$\therefore (F \vee F) \wedge F \quad \Rightarrow T$

$\therefore F \wedge F \Rightarrow T \qquad \therefore F \Rightarrow T$

Q.38 (A)

$\vec{a} = (1,1,4) = \hat{i} + \hat{j} + 4\hat{k}, \vec{b} = (1,-1,4) = \hat{i} - \hat{j} + 4\hat{k}$

$\therefore \vec{a} + \vec{b} = 2\hat{i} + 8\hat{k} \Rightarrow \vec{a} - \vec{b} = 2\hat{j}$

Since, $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$

$\therefore (a+b) \perp (a-b)$.

Hence $\theta = 90°$.

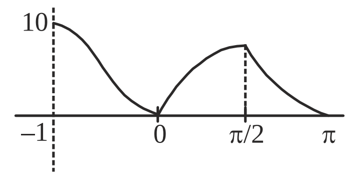
Q.39 (C)

$$\int e^{2\ln(x^2+1)} dx = \int e^{\ln(x^2+1)^2} dx = \int (x^2+1)^2 dx$$

$$= \int (x^4 + 2x^2 + 1) dx = \frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C \;]$$

Q.40 (B)

$$f'(x) = \begin{cases} 3x^2 + 2x - 10; & -1 < x < 0 \\ \cos x; & 0 < x < \frac{\pi}{2} \\ -\sin x; & \frac{\pi}{2} < x < \pi \end{cases}$$



so at $x = \frac{\pi}{2}$, local maxima

by graph at $x = 0, \pi$ absolute minima and at $x = -1$ absolute maxima

Q.41 (B)

$$\frac{dy}{dx} = \frac{2}{x^2} \Rightarrow dy = \frac{2}{x^2}dx \Rightarrow y = -\frac{2}{x} + c$$

Q.42 (D)

$$I = \int \frac{3e^x + 5e^{-x}}{4e^x - 5e^{-x}} dx = \int \frac{3e^{2x}+5}{4e^{2x}-5} dx$$

Let $3e^{2x} + 5 = A(4e^{2x} - 5) + B(8e^{2x})$

$\therefore \qquad 4A + 8B = 3 \qquad$ ....(1)

and $\qquad -5A = 5 \Rightarrow A = -1 \qquad$ ....(2)

$\therefore \qquad B = \frac{7}{8}$

$\therefore I = Ax + B\ln|4e^{2x} - 5| + C$ when $A = -1$ and $B = \frac{7}{8}$

Q.43 (B)

$$I = \int_{-1}^{1} \frac{dx}{(1+e^x)(1+x^2)} \quad ....(1)$$

$$= \int_{-1}^{1} \frac{dx}{1+e^{-x}} \cdot \frac{1}{1+x^2} \qquad \text{(using King)}$$

$$I = \int_{-1}^{1} \frac{e^x \; dx}{(1+e^x)(1+x^2)} \quad ....(2)$$

adding (1) and (2)

$$2I = \int_{-1}^{1} \frac{(1+e^x)\,dx}{(1+e^x)(1+x^2)} = \int_{-1}^{1} \frac{dx}{(1+x^2)}$$

$$= 2\int_0^1 \frac{dx}{(1+x^2)} \Rightarrow I = \int_0^1 \frac{dx}{(1+x^2)} = \tan^{-1}(1) = \pi/4$$

Q.44 (C)

$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = -\hat{i} + 4\hat{j}$

Area of quadrilateral BDCE

$$= \frac{1}{2}\left|\overrightarrow{BC} \times \overrightarrow{DE}\right| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 0 \\ 4 & -2 & 0 \end{vmatrix} = \frac{1}{2}\left|-14\hat{k}\right| = 7$$

Q.45 (A)

$$\frac{dV(t)}{dt} = -k(T-t) \Rightarrow \int dV = \int -k\,(T-t)dt \Rightarrow$$

$$V(t) = \frac{-k(T-t)^2}{-2} + C$$

$$\Rightarrow V(t) = \frac{k(T-t)^2}{2} + C$$

At $t = 0, V = I \Rightarrow C = I - \frac{kT^2}{2}$

$\therefore$ Scrap value V(T) $\Rightarrow V(t = T) = C = I - \frac{kT^2}{2}$.

Q.46 (B)

E = x is a prime number

$P(E) = P(2) + P(3) + P(5) + P(7) = 0.62$

$F = (x < 4), P(F) = P(1) + P(2) + P(3) = 0.50$

$\therefore P(E \cap F) = P(E) + P(F) - P(E \cup F)$

$= 0.62 + 0.50 - 0.35 = 0.77$

Q.47 (B)

$\because$ P is a p.m.f. $\therefore \sum_{z=1}^{5} P(X = x) = 1$

$\therefore (X = 1) + P(X = 2) + ...+ P(X = 5) = 1$

$\therefore k(1) + k(2) + k(3) + k(4) + k(5) = 1$

$\therefore 15k = 1 \quad \therefore k = \frac{1}{15}$

Q.48 (A)

The repeated tosses of a coin are bernoulli trials. Let X denotes the number of heads in an experiment of 10 trials.

Clearly, X has the binomial distribution with n = 10 and $p = \frac{1}{2}$

Therefore $P(X = x) = {}^nC_x q^{n-x} p^x$,

$x = 0, 1, 2, ..., n$

Here $n = 10, p = \frac{1}{2}, q = 1 - p = \frac{1}{2}$

Therefore

$$P(X = x) = {}^{10}C_x \left(\frac{1}{2}\right)^{10-x}\left(\frac{1}{2}\right)^x = {}^{10}C_x \left(\frac{1}{2}\right)^{10}$$

Now $P(X = 6) = {}^{10}C_6 \left(\frac{1}{2}\right)^{10} = \frac{10!}{6! \times 4! 2^{10}} = \frac{105}{512}$

Q.49 (D)

$$\frac{f(3) - f(1)}{3-1} = f'(c);$$

hence $f'(c) = +\frac{1}{c^2} = \frac{\frac{2}{3} - 0}{2} = \frac{1}{3} \Rightarrow c = \sqrt{3}$

Q.50 (B)

$$\frac{dx}{dy} = \frac{x^2 + y^2}{xy}$$

Put $x = uy$

$$\frac{dx}{dy} = u + y\frac{du}{dy} \Rightarrow u + y\frac{du}{dy} = \frac{u^2 + 1}{u}$$

$$\Rightarrow u + y\frac{du}{dy} = u + \frac{1}{u}$$

$$\int u\,du = \int \frac{dy}{y}$$

$$\frac{u^2}{2} = \log y + C \Rightarrow \frac{x^2}{2y^2} = \log y + C$$

Curve passes through (1, 1) $\Rightarrow C = \frac{1}{2}$

Also, $y(k) = \sqrt{e}$

$$\Rightarrow \frac{k^2}{2e} = \log\sqrt{e} + \frac{1}{2} \Rightarrow \frac{k^2}{2e} = 1 \Rightarrow k^2 = 2e$$