KCET Full Test-01_Mathematics Only Solutions

Q.1 (A)

Sol. $x^2 = 16 \Rightarrow x = \pm 4$

$2x = 6 \Rightarrow x = 3$

There is no value of x which satisfies both the above equations. Thus, $A = \phi$

Q.2 (A)

Sol. $A = \{5, 9, 13, 17, 21\}$ and

$B = \{3, 6, 9, 12, 15, 18, 21, 24\}$

$A - B = \{5, 13, 17\}$

$A - (A - B) = \{9, 21\}$

Q.3 (C)

Sol. $\because\ f(x) = \cos^4 x + 3(1 - \cos^2 x)$

$= \cos^4 x - 3\cos^2 x + 3 = \left(\cos^2 x - \frac{3}{2}\right)^2 + \frac{3}{4}$

$\because\ 0 \le \cos^2 x \le 1 \ \Rightarrow\ \frac{-3}{2} \le \cos^2 x - \frac{3}{2} \le \frac{-1}{2}$

$\Rightarrow\ \frac{1}{4} \le \left(\cos^2 x - \frac{3}{2}\right)^2 \le \frac{9}{4}$

$\Rightarrow\ 1 \le \left(\cos^2 x - \frac{3}{2}\right)^2 + \frac{3}{4} \le 3$

$\therefore$ Number of integers in the range = 3

Q.4 (A)

Sol. Since, $(1, 2) \in S$ but $(2, 1) \notin S$

So, S is not symmetric

Hence, S is not an equivalence relation.

Given, $T = \{x, y) : (x - y) \in I\}$

Now, $x - x = 0 \in I$, it is reflexive relation.

Again now, $(x - y) \in I$

$y - x \in I$, it is symmetric relation.

Let $x - y = I_1$ and $y - z = I_2$

Now $x - z = (x - y) + (y - z) = I_1 + I_2 \in I$

So, T is also transitive.

Hence, T is an equivalence relation.

Q.5 (B)

Sol. $\theta = \frac{\pi}{8}$

$E = \cos^4\theta + \cos^4 3\theta + \cos^4 5\theta + \cos^4 7\theta$

$\pi = 8\theta;\ 7\theta = \pi - \theta$

$= 2.[\cos^4\theta + \cos^4 3\theta] \Rightarrow 3\theta = \frac{\pi}{2} - \theta$

$= 2.[\cos^4\theta + \sin^4\theta]$

$= 2.[1 - 2\sin^2\theta\cos^2\theta]$

$= 2.[1 - \frac{1}{2}\sin^2 2\theta]$

$= 2.\left[1 - \frac{1}{2}\cdot\frac{1}{2}\right] = \frac{3}{2}$.

Q.6 (2)

Sol. $N = (a + ib)^3 - 107i$

$= a^3 + 3a^2bi + 3ab^2i^2 + b^3i^3 - 107i$

$= (a^3 - 3ab^2) + i(3a^2b - b^3 - 107)$

since N is + ve real hence its imaginary part = zero

$b(3a^2 - b^2) = 107$

since 107 is prime, hence it has exactly 2 divisions which are 1 and 107

hence $b = 1$ and $3a^2 - b^2 = 107$

if $3a^2 - b^2 = 1$ and $b = 107$ then a is not an integer.

$\therefore\ 3a^2 - b^2 = 107 \Rightarrow 3a^2 = 108 \Rightarrow a^2 = 36$

$\Rightarrow a = 6$ ($a \neq -6$ as it is given)

$\therefore\ N = 216 - 3\cdot 6\cdot 1 = 198$ **Ans.**

Q.7 (D)

Sol.: Total words, without any restriction = 7!

Total words beginning with I = 6!

Total words ending with B = 6!

Total words with beginning I and ending with B = 5!

Thus, total number of required words

$= 7! - (6! + 6! - 5!) = 7! - 2(6!) + 5!$ **Ans.**

Q.8 (C)

Sol. $\because\ {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore\ {}^{10}C_x = {}^{10}C_y = {}^{10}C_{10-y} \Rightarrow x + y = 10$

Q.9 (B)

Sol. $\left(\frac{\sum_{r=1}^{n} 8r^3}{\sum_{r=1}^{n} 27r^3}\right)^{\frac{1}{3}} = \left(\frac{8}{27}\right)^{1/3} = \frac{2}{3}$. **Ans.**

Q.10 (A)

Sol. Given equation is $x^2 = -8ay$. here $A = 2a$

Focus of parabola $(0, -A)$ i.e. $(0, -2a)$

Directrix $y = A$ i.e., $y = 2a$

Q.11 (B)

Sol. Here the lines are, $3x + 4y - 9 = 0$ .....(i)

and $6x + 8y - 15 = 0$ .....(ii)

Now distance from origin of both the lines are

$\frac{-9}{\sqrt{3^2 + 4^2}} = \frac{9}{5}$ and $\frac{-15}{\sqrt{6^2 + 8^2}} = -\frac{15}{10}$

Hence distance between both the lines are

$\left|-\frac{9}{5} - \left(-\frac{15}{10}\right)\right| = \frac{3}{10}$.

Aliter: Put y = 0 in the first equation, we get x = 3 therefore, the point (3, 0) lies on it. So the required distance between these two lines is the perpendicular length of the line $6x + 8y = 15$ from the point (3, 0). i.e., $\frac{6\times 3 - 15}{\sqrt{6^2 + 8^2}} = \frac{3}{10}$.

KCET Full Test-01_Mathematics Only Solutions

Q.12 (D)

Sol. $\frac{-4+1}{5+4}=\frac{2-3}{5-2}=\frac{-2-2}{\lambda+2}$ or $\lambda + 2 = 12$ or $\lambda = 10$.

Q.13 (A)

Sol.: $\frac{0}{0}$ form

Applying L'Hospital's rule

$$\text{limit} = \lim_{x\to a}\frac{\frac{5}{3}(x+2)^{\frac{2}{3}}}{1} = \frac{5}{3}(a+2)^{\frac{2}{3}}$$

Q.14 (C)

Sol. Total number of observation are 9 which is odd and it means median is 5th item. Now we are increasing 2 in the last four items which does not effect its value. So, new median remains unchanged.

Q.15 (C)

Sol. $f(x)=1+\frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$

$\Rightarrow$ Range = (1, 7/3]

Q.16 (D)

Sol. $\cos(\sin x) \ge 0\ \forall\ x \in R \quad (\because -1 \le \sin x \le 1)$

$\& \ -1 \le \frac{1+x^2}{2x} \le 1.$

If $x < 0$, then $\frac{1+x^2}{2x} < 0$ i.e. $-1 \le \frac{1+x^2}{2x}$

$\Rightarrow -2x \ge (1 + x^2) \Rightarrow (1 + x^2) \le 0$

$\Rightarrow x = -1$

If $x > 0$,

$1 + x^2 \le 2x$

$\Rightarrow (x-1)^2 \le 0$

$\Rightarrow x = 1.$

The function is defined only for the two values x = 1 and x = – 1.

Q.17 (C)

Sol.: $\because\ A(\text{adj } A) = |A|\, I_n \Rightarrow |A| = 4$

$$\frac{|\text{adj}(\text{adj}A)|}{|\text{adj}A|} = \frac{|A|^{(3-1)^2}}{|A|^{3-1}} = \frac{4^4}{4^2} = 4^2 = 16 \ \textbf{Ans.}$$

Q.18 (B)

Sol.: Directly open by R_1 to get

$\cos^2(\theta + \phi) + \sin^2(\theta + \phi) + \cos 2\phi$

$= 1 + \cos 2\phi$. Which is independent of θ

Q.19 (B)

Sol. For continuous function

$f(0^-) = f(0) = f(0^+)$

$f(0^-) = (P + 1) + 1 = P + 2\ \&\ f(0) = q$

$$f(0^+) = \lim_{x\to 0^+}\frac{\sqrt{1+x}-1}{x} = \lim_{x\to 0^+}\frac{1}{\sqrt{1-x}+1} = \frac{1}{2}$$

$q = \frac{1}{2};\ P + 2 = q \quad \Rightarrow \ P = \frac{-3}{2}$

Q.20 (D)

Sol.: Since $\lim_{x\to 1/2} f(x) \ne f\left(\frac{1}{2}\right)$.

Q.21 (D)

Sol. Given, $V = \pi r^2 h$

Differentiating both sides

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2r\frac{dr}{dt}h\right) = \pi r\left(r\frac{dh}{dt} + 2h\frac{dr}{dt}\right)$$

$$\frac{dr}{dt} = \frac{1}{10} \quad \text{and} \quad \frac{dh}{dt} = -\frac{2}{10}$$

$$\frac{dV}{dt} = \pi r\left(r\left(-\frac{2}{10}\right) + 2h\left(\frac{1}{10}\right)\right) = \frac{\pi r}{5}(-r+h)$$

Thus, when r = 2 and h = 3,

$$\frac{dV}{dt} = \frac{\pi(2)}{5}(-2+3) = \frac{2\pi}{5}$$

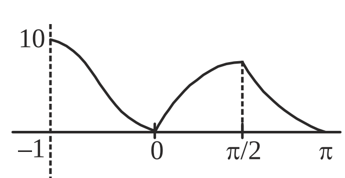
Q.22 (D)

Sol.: $\frac{d}{dx}\left[\frac{\cot^2 x - 1}{\cot^2 x + 1}\right] = \frac{d}{dx}\left[\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}\right]$

$= \frac{d}{dx}[\cos 2x] = -2\sin 2x$

Q.23 (B)

Sol.:
$$f'(x) = \begin{cases} 3x^2 + 2x - 10; & -1 < x < 0 \\ \cos x; & 0 < x < \frac{\pi}{2} \\ -\sin x; & \frac{\pi}{2} < x < \pi \end{cases}$$



so at $x = \frac{\pi}{2}$, local maxima

by graph at $x = 0, \pi$ absolute minima and at $x = -1$ absolute maxima

KCET Full Test-01_Mathematics Only Solutions

Q.24 (A)

Sol.: $$I = 4\int_0^{\pi/2} \frac{dx}{\sin^2 x + k^2\cos^2 x} \quad \text{where } k = 2008$$

$$= 4\int_0^{\pi/2} \frac{\cos^2 x\, dx}{\tan^2 x + k^2} \quad (\text{put } \tan x = t)$$

$$= 4\int_0^{\infty} \frac{dt}{t^2 + k^2} = \frac{4}{k}\tan^{-1}\frac{t}{k}\Big]_0^{\infty} = \frac{4}{k}\cdot\frac{\pi}{2} = \frac{2\pi}{k}$$

$$\therefore I = \frac{2\pi}{2008} = \frac{\pi}{1004}$$

$\therefore a + b = 1005$ **Ans.**

Q.25 (B)

Sol.: $$\left.\begin{array}{l} |3\vec{a} + 4\vec{b}|^2 = 9\lambda^2 + 16\lambda^2 = 25\lambda^2 \\ |4\vec{a} + 3\vec{b}|^2 = 16\lambda^2 + 9\lambda^2 = 25\lambda^2 \end{array}\right]$$

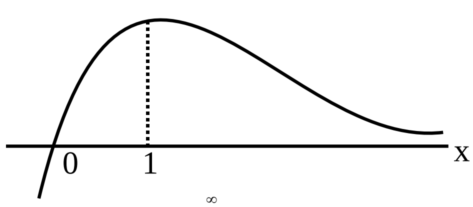
[Let $\lambda = |\vec{a}| = |\vec{b}|$ (say)]

given, $5\lambda + 5\lambda = 20 \Rightarrow \lambda = 2 \Rightarrow |\vec{a}| = |\vec{b}| = 2$

$$\Rightarrow |(\vec{a}\times\vec{b})\times\vec{a}| = |\vec{a}\times\vec{b}|\,|\vec{a}|\sin 90^\circ = |\vec{a}|^2 \times |\vec{b}| = 8$$

Q.26 (A)

Sol.



Required area $A = \int_0^{\infty} e^{-x}x = -xe^{-x} - e^{-x}\Big|_0^{\infty} = 1$

Q.27 (B)

Sol.: put $xe^x = t$

$$\Rightarrow (e^x + xe^x)\frac{dx}{dy} = \frac{dt}{dy}$$

$$\therefore \frac{dt}{dy} + (ye^y - t) = 0 \Rightarrow \frac{dt}{dy} - t + ye^y = 0$$

I.F. $e^{-\int dy} = e^{-y}$

$$t \cdot e^{-y} = -\int ye^y e^{-y}dy$$

$$x\,e^x e^{-y} = -\frac{y^2}{2} + C$$

$f(0) = 0 \Rightarrow C = 0;\quad 2x\,e^x e^{-y} + y^2 = 0$ Ans.

Q.28 (D)

Sol.: $$\int(\sin 2x - \cos 2x)dx = -\frac{1}{2}(\sin 2x + \cos 2x) + C$$

$$\Rightarrow -\left[\frac{1}{\sqrt{2}}\sin 2x + \frac{1}{\sqrt{2}}\cos 2x\right] + C$$

$$\Rightarrow -\frac{1}{\sqrt{2}}\sin\left(2x + \frac{\pi}{4}\right) + C = \frac{1}{\sqrt{2}}\sin\left(2x + \frac{5\pi}{4}\right) + C$$

$\Rightarrow$ b is any constant and $a = \frac{-5\pi}{4}$.

Q.29 (D)

Sol.: Let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$

Now, $0 = \vec{c}\cdot\vec{b} = \lambda\,\vec{a}\cdot\vec{b} + \mu\vec{b}^2 = -\lambda + 5\mu$ ....(A)

Again, $7 = \vec{a}\cdot\vec{c} = \lambda\,\vec{a}^2 + \mu\vec{a}\cdot\vec{b} = 3\lambda - \mu$ ......(B)

$\therefore$ Solving (A) and (B), we get

$$\lambda = \frac{5}{2}, \mu = \frac{1}{2}$$

Thus, $\vec{c} = \frac{5}{2}\left(-\hat{i} + \hat{j} + \hat{k}\right) + \frac{1}{2}\left(2\hat{i} + \hat{k}\right)$

$$= \frac{-3}{2}\hat{i} + \frac{5}{2}\hat{j} + 3\hat{k} \Rightarrow 2|\vec{c}|^2 = 35.$$ **Ans.**

Q.30 (B)

Sol.: Shortest distance

$$= \left|\frac{(-2\hat{i} + 2\hat{j} + \hat{k})\cdot\left((\hat{i} + 2\hat{j} + 3\hat{k})\times(-\hat{i} + 3\hat{j} + 7\hat{k})\right)}{|(\hat{i} + 2\hat{j} + 3\hat{k})\times(-\hat{i} + 3\hat{j} + 7\hat{k})|}\right|$$

$$= \frac{5}{\sqrt{6}} = \frac{5\sqrt{6}}{6}$$ **Ans.**

Q.31 (A)

Sol. $f'(x) = 2 + \cos x > 0$. So, $f(x)$ is strictly monotonic increasing so, $f(x)$ is one-to-one and onto.

Q.32 (D)

Sol. E $\rightarrow$ Lost card not spade

A $\rightarrow$ both cards drawn are spade

$P(E) = \frac{3}{4}$ $\qquad P(E^C) = \frac{1}{4}$

$P\left(\frac{A}{E}\right) = \frac{^{13}C_2}{^{51}C_2}$ $\qquad P\left(\frac{A}{E^C}\right) = \frac{^{12}C_2}{^{52}C_2}$

$$P\left(\frac{A}{E}\right) = \frac{P(E)P\left(\frac{A}{E}\right)}{P(E)P\left(\frac{A}{E}\right) + P(E^C).P\left(\frac{A}{E^C}\right)} = \frac{39}{50}$$

KCET Full Test-01_Mathematics Only Solutions

Q.33 (A)

Sol.: $\frac{5(-8+6i)}{(1+i)^2} = a + ib$

$\Rightarrow \frac{-40+30i}{2i} = 15 + 20i = a + ib$

Equating real and imaginary parts,
we get a = 15 and b = 20.

Q.34 (A)

Sol. $2\sin^2 2\theta = b^{5/4}$ ......(A)

also $5\sin^2\theta = 2 + \cos^2\theta = 3 - \sin^2\theta$

$6\sin^2\theta = 3 \Rightarrow \sin^2\theta = \frac{1}{2}$ $\quad \therefore \cos^2\theta = \frac{1}{2}$

$8\sin^2\theta\ \cos^2\theta = b^{5/4}$

$\Rightarrow 8 \cdot \frac{1}{2}\left(\frac{1}{2}\right) = b^{5/4} \Rightarrow 2 = b^{5/4}$

$\therefore b = (B)^{4/5} \Rightarrow q = \frac{4}{5}$.

Q.35 (D)

Sol. Required number of ways
= Total when all A's seperated
– Total when A's separated and H's are together

$= \frac{7!}{2!}({}^8C_4) - 6!({}^7C_4)$

$= \frac{7!6!}{4!.3!}(6) = 4^1.5^2.6^3.7^1$

Q.36 (A)

Sol. $N = (1+5)^{11} - 55$

$N = (1 + 11 \times 5 + \text{multiple of } 25) - 55$

$N = 1 + \text{multiple of } 25$

$\Rightarrow$ Remainder is 1

Q.37 (A)

Sol. $5\left(-\frac{1}{2}\right)^{n-1} = \frac{5}{1024}$

$\Rightarrow \left(\frac{-1}{2}\right)^{10} = \left(\frac{-1}{2}\right)^{n-1} \Rightarrow 10 = n - 1 \Rightarrow n = 11$

Q.38 (D)

Sol.: We have $f(x) = \frac{3x^2 + ax + a + 1}{(x+2)(x-1)}$

$\therefore 12 - 2a + a + 1 = 0 \Rightarrow a = 13$

Now $\lim_{x\to -2} \frac{3x^2 + 13x + 14}{(x+2)(x-1)}$

$= \lim_{x\to -2} \frac{(3x+7)(x+2)}{(x+2)(x-1)} = \frac{-1}{3}$

Q.39 (A)

Sol. $\bar{x} = \frac{\text{sum of quantities}}{n} = \frac{\frac{n}{2}(a+1)}{n}$

$= \frac{1}{2}[1 + 1 + 100d] = 1 + 50d \ \therefore\ MD = \frac{1}{n}\Sigma|x_i - \bar{x}|$

$\Rightarrow 255 = \frac{1}{101}[50d + 49d + ... + d + 0 + d + 49d + 50d]$

$= \frac{2d}{101}\left[\frac{50\times 51}{2}\right] \Rightarrow d = \frac{255\times 101}{50\times 51} = 10.1$

Q.40 (C)

Sol.: x = {1, 2, 3, ...
If x = 2n, n ∈ N then f(x) = x – 1
f(x) ∈ {1, 3 ....}
If x = 2n – 1, n ∈ N then f(x) = x + 1
then f(x) ∈ {2, 4, ...]

Q.41 (A)

Sol. $\begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0 \Rightarrow k = -5, \frac{5}{2}(\text{reject})$

$\Rightarrow$ Angle between lines given by

$\cos\theta = \left|\frac{3k + 2k + 6}{k^2 + 13}\right| = \frac{1}{2}$

Q.42 (A)

Sol. $\because\ \text{cosec}^{-1} x + \sec^{-1} x^2 + \frac{\pi}{2} = 0$

$\Rightarrow \text{cosec}^{-1} x = -\frac{\pi}{2}$ and $\sec^{-1} x^2 = 0$

$\Rightarrow x = -1$ and $x^2 = 1 \Rightarrow x = -1$

$\therefore \alpha = -1$

$\therefore \sin^{-1}\alpha - \cos^{-1}\alpha = -\frac{\pi}{2} - \pi = -\frac{3\pi}{2}$

Q.43 (C)

Sol. $AB = \begin{bmatrix} -x & -y & 7z \\ 0 & y & 14z \\ x & -y & 7z \end{bmatrix}$

equating coefficients $x^2 = 4$; $y^2 = 9$ and $z^2 = 1$

$\Rightarrow x = \pm 2,\ y = \pm 3$ and $z = \pm 1$

number of solution = 2 × 2 × 2 = 8

Q.44 *(B)*

Sol.: $\Delta \equiv \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ 2\alpha + b & b\alpha + c & 0 \end{vmatrix}$

by expanding along R_3

$= (b^2 - ac)(a\alpha^2 + 2b\alpha + c)$

KCET Full Test-01_Mathematics Only Solutions

Q.45 (C)

Sol.: $f(x)=\begin{cases} -x\sin x+(x^2-\pi^2)\cos x, & x<-\pi \\ -x\sin x+(\pi^2-x^2)\cos x, & -\pi<x<0 \\ x\sin x+(\pi^2-x^2)\cos x, & 0\le x<\pi \\ x\sin x+(x^2-\pi^2)\cos x, & \pi\le x \end{cases}$

$\Rightarrow y = f(x)$ is not differentiable at $x = \pi$ or $-\pi$.

Q.46 (C)

Sol.: $f(x)$ possesses derivative at $x = 0$, so it is both continuous and differentiable at $x = 0$. Now
$f(0 + 0) = 0, f(0 - 0) = b, f(0) = b, \therefore b = 0$
Also $Rf'(0) = 0, Lf'(0) = 0, \forall a \in R$
$\therefore f'(0) = 0$ if $b = 0$

Q.47 (D)

Sol.: Since $\frac{dy}{dx} = -\sin(\sin x^2).\cos x^2.2x$

Therefore, at $x=\sqrt{\frac{\pi}{2}}, \cos x^2=\cos\frac{\pi}{2}=0 \Rightarrow \frac{dy}{dx}=0.$

Q.48 (A)

Sol. $\because \frac{dr}{dt}=3, \frac{dh}{dt}=-4, \quad r=4, h=6$

$\because v=\pi r^2 h$

$\Rightarrow \frac{dv}{dt}=2\pi rh\frac{dr}{dt}+\pi r^2\frac{dh}{dt}$

$\therefore \frac{dv}{dt}=2\pi\cdot4\cdot6\cdot3-\pi\cdot16\cdot4$

$= 144\pi - 64\pi = 80\pi\ m^3/s.$

Q.49 (C)

Sol.: Put $\sin x = t \Rightarrow f'(t) = 1 - t^2$

$\Rightarrow f(t)=t-\frac{t^3}{3}+c \qquad \because f(0)=1 \Rightarrow c=\frac{1}{3}$

Q.50 (B)

Sol. $I=\int_{-\pi/2}^{\pi/2}\frac{\cos\theta\, d\theta}{(2-\sin\theta)\cos\theta}$ (putting $x = \sin\theta$)

$=\int_0^{\pi/2}\left(\frac{1}{2-\sin\theta}+\frac{1}{2+\sin\theta}\right)d\theta$

$\left[\text{using } \int_{-a}^{a} f(x)\,dx=\int_0^a[f(x)+f(-x)]\,dx\right]$

$=4\int_0^{\pi/2}\frac{d\theta}{4-\sin^2\theta}=\frac{4}{3}\int_0^{\pi/2}\frac{\sec^2\theta\, d\theta}{\frac{4}{3}+\tan^2\theta}$

$=\frac{4}{3}\int_0^{\infty}\frac{d\theta}{t^2+\frac{4}{3}}=\frac{4}{3}.\frac{\sqrt3}{2}.\tan^{-1}\frac{\sqrt3 t}{2}\Big|_0^{\infty}=\frac{2}{\sqrt3}.\frac{\pi}{2}=\frac{\pi}{\sqrt3}$

Q.51 (D)

Sol.: $\int\frac{dx}{\sqrt{1+x}+\sqrt{x}}=\int\left[\frac{(x+1)-x}{\sqrt{1+x}+\sqrt{x}}\right]dx$

$\int\left(\sqrt{x+1}-\sqrt{x}\right)dx=\frac{(x+1)^{3/2}}{3/2}-\frac{x^{3/2}}{3/2}+c$

$=\frac{2}{3}\left[(x+1)^{3/2}-x^{3/2}\right]+c=\frac{2}{3}(x+1)^{3/2}-\frac{2}{3}x^{3/2}+c$

Q.52 (C)

Sol.: Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$, then

$\int\frac{dx}{(x^2-1)\sqrt{x^2+1}}=\int\frac{\sec^2\theta d\theta}{(\tan^2\theta-1)\sec\theta}=\int\frac{\cos\theta d\theta}{(2\sin^2\theta-1)}$

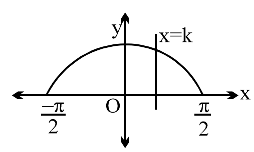
Again put $t = \sin\theta \Rightarrow dt = \cos\theta\, d\theta$, then it reduces to

$\int\frac{dt}{(2t^2-1)}=\frac{1}{2}\int\frac{dt}{t^2-\left(\frac{1}{\sqrt2}\right)^2}=\frac{1}{2\sqrt2}\log\left(\frac{t-\frac{1}{\sqrt2}}{t+\frac{1}{\sqrt2}}\right)+c$

$=\frac{1}{2\sqrt2}\log\left(\frac{\sqrt{1+x^2}-x\sqrt2}{\sqrt{1+x^2}+x\sqrt2}\right)+c$

Q.53 (C)

Sol.



$\int_{-\pi/2}^{k}\cos x\,dx=3\int_k^{\pi/2}\cos x\,dx$;

$\sin k-\sin\left(-\frac{\pi}{2}\right)=3\left(\sin\frac{\pi}{2}-\sin k\right)$

$\sin k + 1 = 3 - 3\sin k$; $4\sin k = 2$

$\Rightarrow k=\frac{\pi}{6}.$

Q.54 (B)

Sol. $\frac{dx}{dy}=\frac{x^2+y^2}{xy}$

Put $x = uy$

$\frac{dx}{dy}=u+y\frac{du}{dy} \Rightarrow u+y\frac{du}{dy}=\frac{u^2+1}{u}$

$\Rightarrow u+y\frac{du}{dy}=u+\frac{1}{u}$

KCET Full Test-01_Mathematics Only Solutions

$$\int u\,du = \int \frac{dy}{y}$$

$$\frac{u^2}{2} = \log y + C \Rightarrow \frac{x^2}{2y^2} = \log y + C$$

Curve passes through (1, 1) $\Rightarrow C = \frac{1}{2}$

Also, $y(k) = \sqrt{e}$

$$\Rightarrow \frac{k^2}{2e} = \log\sqrt{e} + \frac{1}{2} \Rightarrow \frac{k^2}{2e} = 1 \Rightarrow k^2 = 2e$$

Q.55 (A)

Sol. $(\vec{a}-\vec{b})\times\vec{c} = \vec{0} \Rightarrow \vec{c} \parallel (\vec{a}-\vec{b})$

$\Rightarrow \vec{c} = t(-2\hat{i} + 7\hat{j} + 2\lambda\hat{k})$

$\because \bar{a}\cdot\bar{c} = 7$

$\Rightarrow t(-2 + 14 + 2\lambda^2) = 7$

$\Rightarrow t(2\lambda^2 + 12) = 7$ ...(i)

$\because \bar{b}.\bar{c} = \frac{-43}{2}$

$\Rightarrow t(-6 - 35 - 2\lambda^2) = -\frac{43}{2}$ ...(ii)

From (i) & (ii)

$$\left(\frac{7}{2\lambda^2+12}\right)\left(-41-2\lambda^2\right) = -\frac{43}{2}$$

$\Rightarrow (287 + 14\lambda^2) = 43(\lambda^2 + 6)$

$287 + 14\lambda^2 = 43\lambda^2 + 258 \Rightarrow 29\lambda^2 = 29$

$\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = 1$

Now $|\bar{a}.\bar{b}| = |3 - 10 - \lambda^2| = |-7-1| = 8$

Q.56 (D)

Sol. Given, position vectors of A, B and C are

$7\hat{j} + 10\hat{k}, -\hat{i} + 6\hat{j} + 6\hat{k}$ and $-4\hat{i} + 9\hat{j} + 6\hat{k}$ respectively.

$\therefore |\overrightarrow{AB}| = |-\hat{i} - \hat{j} - 4\hat{k}| = \sqrt{18}$

$|\overrightarrow{BC}| = |-3\hat{i} + 3\hat{j}| = \sqrt{18}$

$|\overrightarrow{AC}| = |-4\hat{i} + 2\hat{j} - 4\hat{k}| = \sqrt{36}$

Clearly, AB = BC and $(AC)^2 = (AB)^2 + (BC)^2$

Hence, triangle is right angled isosceles.

Q.57 (D)

Sol.: (1, 2, 3) lie on

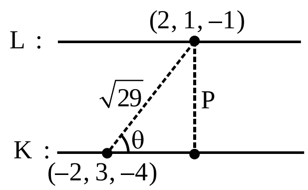
$L \therefore -\frac{1}{2} = \frac{1}{b} = \frac{4}{c}$

$\therefore b = -2, c = -8$

Line L and K are parallel

$\frac{2}{a} = \frac{b}{2} = \frac{c}{d} \Rightarrow \frac{2}{a} = -1 = -\frac{8}{d}$

$\therefore a = -2, d = 8$



$L: \frac{x-2}{2} = \frac{y-1}{-2} = \frac{z+1}{-8}$

$K: \frac{x+2}{-2} = \frac{y-3}{2} = \frac{z+4}{8}$

$$\cos\theta = \left|\frac{8+4-24}{\sqrt{72}\sqrt{29}}\right| = \frac{\sqrt{2}}{\sqrt{29}}$$

Q.58 (B)

Sol.: Distance from x-axis $= \sqrt{y^2 + z^2} = \sqrt{(b^2 + c^2)}$

Q.59 (C)

Sol. Since there are one A, two I and one O, hence the required probability $= \frac{1+2+1}{11} = \frac{4}{11}$

Q.60 (B)

Sol. 52 < 40 non face card, 12 face card

Consider events

F_1 : first card is face card

F_2 : second card is face card

$P(A) = 1 - P(F_1 \cap F_2)$

$$\Rightarrow p_1 = 1 - \frac{12}{52}\cdot\frac{11}{51} = 1 - \frac{11}{13\cdot 17} = \frac{221-11}{221} = \frac{210}{221}$$

$$P(B) = P\left(F_1 \cap \bar{F}_2 \text{ or } \bar{F}_1 \cap \bar{F}_2\right)$$

$$= \frac{12}{52}\cdot\frac{40}{51} + \frac{40}{52}\cdot\frac{39}{51}$$

$$p_2 = \frac{40}{221} + \frac{130}{221} = \frac{170}{221}$$ **Ans.**