JEE-MAIN EXAMINATION-JANUARY 2025

Date: 29/01/2025 ***Shift : Morning***

PHYSICS

SECTION-A

Q.1 Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Choke coil is simply a coil having a large inductance but a small resistance. Choke coils are used with fluorescent mercury-tube fittings. If household electric power is directly connected to a mercury tube, the tube will be damaged.

Reason (R) : By using the choke coil, the voltage across the tube is reduced by a factor $\left(R/\sqrt{R^2+\omega^2L^2}\right)$, where ω is frequency of the supply across resistor R and inductor L. If the choke coil were not used, the voltage across the resistor would be the same as the applied voltage.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(2) (A) is false but (R) is true.

(3) Both (A) and (R) are true and (R) is the correct explanation of (A).

(4) (A) is true but (R) is false.

Q.2 Two projectiles are fired with same initial speed from same point on ground at angles of $(45°-\alpha)$ and $(45°+\alpha)$, respectively, with the horizontal direction. The ratio of their maximum heights attained is :

(1) $\frac{1-\tan\alpha}{1+\tan\alpha}$ (2) $\frac{1+\sin\alpha}{1-\sin\alpha}$

(3) $\frac{1-\sin2\alpha}{1+\sin2\alpha}$ (4) $\frac{1+\sin2\alpha}{1-\sin2\alpha}$

Q.3 An electric dipole of mass m, charge q, and length l is placed in a uniform electric field $\vec{E}=E_0\hat{i}$. When the dipole is rotated slightly from its equilibrium position and released, the time period of its oscillations will be :

(1) $\frac{1}{2\pi}\sqrt{\frac{2\,ml}{qE_0}}$ (2) $2\pi\sqrt{\frac{ml}{qE_0}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{ml}{2qE_0}}$ (4) $2\pi\sqrt{\frac{ml}{2qE_0}}$

Q.4 The pair of physical quantities not having same dimensions is :

(1) Torque and energy

(2) Surface tension and impulse

(3) Angular momentum and Planck's constant

(4) Pressure and Young's modulus

Q.5 Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Time period of a simple pendulum is longer at the top of a mountain than that at the base of the mountain.

Reason (R) : Time period of a simple pendulum decreases with increasing value of acceleration due to gravity and vice-versa.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(2) Both (A) and (R) are true and (R) is the correct explanation of (A).

(3) (A) is true but (R) is false.

(4) (A) is false but (R) is true.

Q.6 The expression given below shows the variation of velocity (v) with time (t), $v=At^2+\frac{Bt}{C+t}$.

The dimension of ABC is :

(1) $[M^0L^2T^{-3}]$ (2) $[M^0L^1T^{-3}]$

(3) $[M^0L^1T^{-2}]$ (4) $[M^0L^2T^{-2}]$

Q.7 Consider I_1 and I_2 are the currents flowing simultaneously in two nearby coils 1 & 2, respectively. If L_1 = self inductance of coil 1, M_{12} = mutual inductance of coil 1 with respect to coil 2, then the value of induced emf in coil 1 will be

(1) $\varepsilon_1=-L_1\frac{dI_1}{dt}+M_{12}\frac{dI_2}{dt}$

(2) $\varepsilon_1=-L_1\frac{dI_1}{dt}-M_{12}\frac{dI_1}{dt}$

(3) $\varepsilon_1=-L_1\frac{dI_1}{dt}-M_{12}\frac{dI_2}{dt}$

(4) $\varepsilon_1=-L_1\frac{dI_2}{dt}-M_{12}\frac{dI_1}{dt}$

Q.8 At the interface between two materials having refractive indices n_1 and n_2, the critical angle for reflection of an em wave is θ_{1c}. The n_2 material is replaced by another material having refractive index n_3, such that the critical angle at the interface between n_1 and n_3 materials is θ_{2c}. If $n_3 > n_2 > n_1$;; $\frac{n_2}{n_3} = \frac{2}{5}$ and $\sin\theta_{2C} - \sin\theta_{1C} = \frac{1}{2}$, then θ_{1C} is

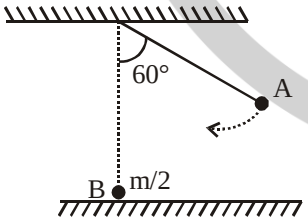
(1) $\sin^{-1}\left(\frac{1}{6n_1}\right)$ (2) $\sin^{-1}\left(\frac{2}{3n_1}\right)$

(3) $\sin^{-1}\left(\frac{5}{6n_1}\right)$ (4) $\sin^{-1}\left(\frac{1}{3n_1}\right)$

Q.9 Consider a long straight wire of a circular cross-section (radius a) carrying a steady current I. The current is uniformly distributed across this cross-section. The distances from the centre of the wire's cross-section at which the magnetic field [inside the wire, outside the wire] is half of the maximum possible magnetic field, any where due to the wire, will be

(1) [a/4, 3a/2] (2) [a/2, 2a]
(3) [a/2, 3a] (4) [a/4, 2a]

Q.10 As shown below, bob A of a pendulum having massless string of length 'R' is released from 60° to the vertical. It hits another bob B of half the mass that is at rest on a friction less table in the centre. Assuming elastic collision, the magnitude of the velocity of bob A after the collision will be (take g as acceleration due to gravity)



(1) $\frac{1}{3}\sqrt{Rg}$ (2) $\sqrt{Rg}$ (3) $\frac{4}{3}\sqrt{Rg}$ (4) $\frac{2}{3}\sqrt{Rg}$

Q.11 Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : Emission of electrons in photoelectric effect can be suppressed by applying a sufficiently negative electron potential to the photoemissive substance.
Reason (R) : A negative electric potential, which stops the emission of electrons from the surface of a photoemissive substance, varies linearly with frequency of incident radiation.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) (A) is false but (R) is true.
(2) (A) is true but (R) is false.
(3) Both (A) and (R) are true and (R) is the correct explanation of (A).
(4) Both (A) and (R) are true but (R) is not the correct explanation of (A).

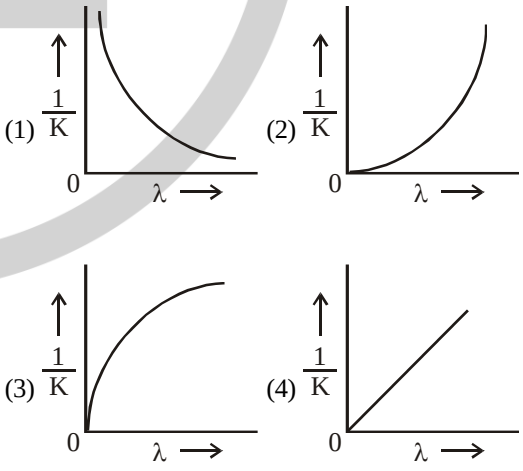
Q.12 A coil of area A and N turns is rotating with angular velocity ω in a uniform magnetic field $\vec{B}$ about an axis perpendicular to $\vec{B}$. Magnetic flux φ and induced emf ε across it, at an instant when $\vec{B}$ is parallel to the plane of coil, are :

(1) $\varphi = AB, \varepsilon = 0$ (2) $\varphi = 0, \varepsilon = NAB\omega$
(3) $\varphi = 0, \varepsilon = 0$ (4) $\varphi = AB, \varepsilon = NAB\omega$

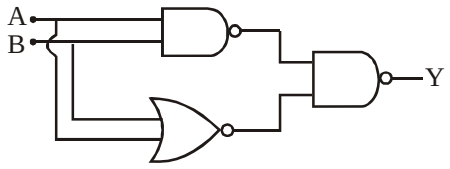
Q.13 The fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the depth of 2.5 km below the sea level is %. Given, the Bulk modulus of water = 2×10^9 Nm^{-2}, density of water = 10^3 kgm^{-3}, acceleration due to gravity g = 10 ms^{-2}.

(1) 1.75 (2) 1.0 (3) 1.5 (4) 1.25

Q.14 If λ and K are de Broglie Wavelength and kinetic energy, respectively, of a particle with constant mass. The correct graphical representation for the particle will be :-



Q.15



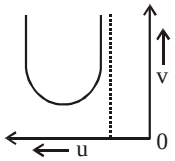
For the circuit shown above, equivalent GATE is :

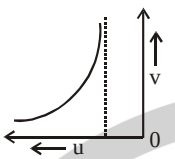
(1) OR gate (2) NOT gate
(3) AND gate (4) NAND gate

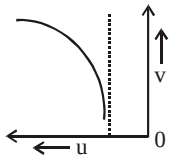
Q.16 A body of mass 'm' connected to a massless and unstretchable string goes in verticle circle of radius 'R' under gravity g. The other end of the string is fixed at the center of circle. If velocity at top of circular path is $n\sqrt{gR}$, where, $n \geq 1$, then ratio of kinetic energy of the body at bottom to that at top of the circle is

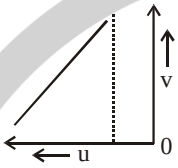
(1) $\frac{n}{n+4}$ (2) $\frac{n+4}{n}$ (3) $\frac{n^2}{n^2+4}$ (4) $\frac{n^2+4}{n^2}$

Q.17 Let u and v be the distances of the object and the image from a lens of focal length f. The correct graphical representation of u and v for a convex lens when |u| > f, is

(1) 

(2) 

(3) 

(4) 

Q.18 Match List-I with List-II.

	List - I		**List- II**
(A)	Electric field inside(distance r > 0 from center) of a uniformly charged spherical shell with surface charge density σ . and radius R.	(I)	$\frac{\sigma}{\varepsilon_0}$
(B)	Electric field at distance r > 0 from a uniformly charged infinite plane sheet with surface charge density σ.	(II)	$\frac{\sigma}{2\varepsilon_0}$
(C)	Electric field outside (distance r > 0 from center) of a uniformly charged spherical shell with surface charge density σ, and radius R	(III)	0
(D)	Electric field between 2 oppositely charged infinite plane parallel sheets with uniform surface charge density σ.	(IV)	$\frac{\sigma}{\varepsilon_0 r^2}$

Choose the correct answer from the options given below :
(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
(2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(3) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Q.19 The work done in an adiabatic change in an ideal gas depends upon only :
(1) change in its pressure (2) change in its specific heat
(3) change in its volume (4) change in its temperature

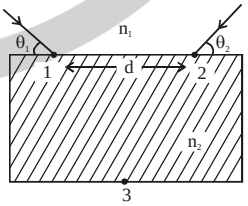
Q.20 Given below are two statements: one is labelled as Assertion (A) and other is labelled as Reason (R).
Assertion (A) : Electromagnetic waves carry energy but not momentum.
Reason (R): Mass of a photon is zero.
In the light of the above statements, choose the most appropriate answer from the options given below :
(1) (A) is true but (R) is false.
(2) (A) is false but (R) is true.
(3) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(4) Both (A) and (R) are true and (R) is the correct explanation of (A).

SECTION-B

Q.21 The coordinates of a particle with respect to origin in a given reference frame is (1, 1, 1) meters. If a force of $\vec{F} = \left(\hat{i} - \hat{j} + \hat{k}\right)$ N-m acts on the particle, then the magnitude of torque (with respect to origin) in z-direction is _____.

Q.22 A container of fixed volume contains a gas at 27°C. To double the pressure of the gas, the temperature of gas should be raised to _____ °C.

Q.23 Two light beams fall on a transparent material block at point 1 and 2 with angle θ_1 and θ_2, respectively, as shown in figure. After refraction, the beams intersect at point 3 which is exactly on the interface at other end of the block. Given : the distance between 1 and 2, $d = 4\sqrt{3}$ cm and $\theta_1 = \theta_2 = \cos^{-1}\left(\frac{n_2}{2n_1}\right)$, where refractive index of the block n_2 > refractive index of the outside medium n_1, then the thickness of the block is _____cm.



Q.24 In a hydraulic lift, the surface area of the input piston is 6 cm^2 and that of the output piston is 1500 cm^2. If 100 N force is applied to the input piston to raise the output piston by 20 cm, then the work done is _____ kJ.

Q.25 The maximum speed of a boat in still water is 27 km/h. Now this boat is moving downstream in a river flowing at 9 km/h. A man in the boat throws a ball vertically upwards with speed of 10 m/s. Range of the ball as observed by an observer at rest on the river bank, is _____cm.

CHEMISTRY
SECTION-A

Q.26 Total number of nucleophiles from the following is :-

NH_3, PhSH, $(H_3C)_2S$, $H_2C=CH_2$, $\overset{\ominus}{O}H$, ,

$H_3O^{\oplus}$ $(CH_3)_2CO$, >= NCH_3

(1) 5 (2) 4 (3) 7 (4) 6

Q.27 The standard reduction potential values of some of the p-block ions are given below. Predict the one with the strongest oxidising capacity.

(1) $E^{\ominus}_{Sn^{4+}/Sn^{2+}} = +1.15$ V (2) $E^{\ominus}_{Tl^{3+}/Tl} = +1.26$ V

(3) $E^{\ominus}_{Al^{3+}/Al} = -1.66$ V (4) $E^{\ominus}_{Pb^{4+}/Pb^{2+}} = +1.67$ V

Q.28 The molar conductivity of a weak electrolyte when plotted against the square root of its concentration, which of the following is expected to be observed?

(1) A small decrease in molar conductivity is observed at infinite dilution.

(2) A small increase in molar conductivity is observed at infinite dilution.

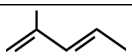
(3) Molar conductivity increases sharply with increase in concentration.

(4) Molar conductivity decreases sharply with increase in concentration.

Q.29 At temperature T, compound $AB_{2(g)}$ dissociates as $AB_{2(g)} \rightleftharpoons AB_{(g)} + \frac{1}{2} B_{2(g)}$ having degree of dissociation x (small compared to unity). The correct expression for x in terms of K_p and p is

(1) $\sqrt[3]{\frac{2K_p}{p}}$ (2) $\sqrt[4]{\frac{2K_p}{p}}$ (3) $\sqrt[3]{\frac{2K_p^2}{p}}$ (4) $\sqrt{K_p}$

Q.30 Match List−I with List−II.

	List− I (Structure)	List− II (IUPAC Name)	
(A)	$H_3C-CH_2-\underset{C_2H_5}{CH}-CH_2-\underset{CH_3}{CH}-C_2H_5$	(I)	4− Methylpent − 1− ene
(B)	$(CH_3)_2C(C_3H_7)_2$	(II)	3− Ethyl− 5− methylheptane
(C)		(III)	4,4− Dimethylheptane
(D)		(IV)	2Methyl− 1, 3 − pentadiene

Choose the correct answer from the options given below:

(1) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(2) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Q.31 Choose the correct statementsÿ

(A) Weight of a substance is the amount of matter present in it

(B) Mass is the force exerted by gravity on an objectÿ

(C) Volume is the amount of space occupied by a substance

(D) Temperatures below 0°C are possible in Celsius scaleÿbut in Kelvin scale negative temperature is not possible

(E) Precision refers to the closeness of various measurements for the same quantity

(1) (B), (C) and (D) Only (2) (A), (B) and (C) Only

(3) (A), (D) and (E) Only (4) (C), (D) and (E) Only

Q.32 correct increasing order of stability of the complexes based on Δ_0 value is

(I) $[Mn(CN)_6]^{3-}$ (II) $[Co(CN)_6]^{4-}$

(III) $[Fe(CN)_6]^{4-}$ (IV) $[Fe(CN)_6]^{3-}$

(1) II < III < I < IV (2) IV < III < II < I

(3) I < II < IV < III (4) III < II < IV < I **[E]**

Q.33 Match List-I with List-II.

List-I (Complex)		List-II (Hybridisation & Magnetic characters)	
(A)	$[MnBr_4]^{2-}$	(I)	d^2sp^3 & diamagnetic
(B)	$[FeF_6]^{3-}$	(II)	sp^3d^2 & paramagnetic
(C)	$[Co(C_2O_4)_3]^{3-}$	(III)	sp^3 & diamagnetic
(D)	$[Ni(CO)_4]$	(IV)	sp^3 & paramagnetic

Choose the correct answer from the options given below :

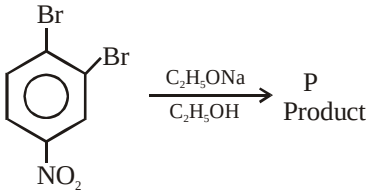
(1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(2) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

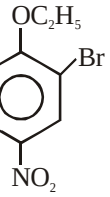
(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

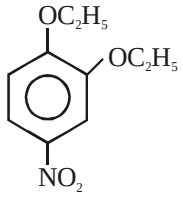
(4) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

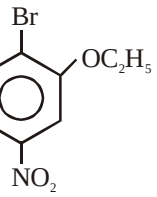
Q.34 In the following substitution reaction :

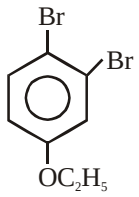


Product 'P' formed is :

(1) 

(2) 

(3) 

(4) 

Q.35 For a Mg | Mg^{2+} (aq) | | Ag^{+} (aq) | Ag the correct Nernst Equation is :

(1) $E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F}\ln\frac{\left[Ag^{+}\right]}{\left[Mg^{2+}\right]}$

(2) $E_{cell} = E^{\circ}_{cell} + \frac{RT}{2F}\ln\frac{\left[Ag^{+}\right]^2}{\left[Mg^{2+}\right]}$

(3) $E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F}\ln\frac{\left[Mg^{2+}\right]}{\left[Ag^{+}\right]}$

(4) $E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F}\ln\frac{\left[Ag^{+}\right]^2}{\left[Mg^{2+}\right]}$

Q.36 The correct option with order of melting points of the pairs (Mn, Fe), (Tc, Ru) and (Re, Os) is :
(1) Fe < Mn, Ru < Tc and Re < Os
(2) Mn < Fe, Tc < Ru and Re < Os
(3) Mn < Fe, Tc < Ru and Os < Re
(4) Fe < Mn, Ru < Tc and Os < Re

Q.37 1.24 g of AX_2 (molar mass 124 g mol^{-1}) is dissolved in 1 kg of water to form a solution with boiling point of 100.0156°C, while 25.4 g of AY_2 (molar mass 250 g mol^{-1}) in 2 kg of water constitutes a solution with a boiling point of 100.0260°C.
$K_b(H_2O)$ = 0.52 K kg mol^{-1}
Which of the following is correct?
(1) AX_2 and AY_2 (both) are completely unionised.
(2) AX_2 and AY_2 (both) are fully ionised.
(3) AX_2 is completely unionised while AY_2 is fully ionised.
(4) AX_2 is fully ionised while AY_2 is completely unionised.

Q.38 500 J of energy is transferred as heat to 0.5 mol of Argon gas at 298 K and 1.00 atm. The final temperature and the change in internal energy respectively are :
Given : R = 8.3 J K^{-1} mol^{-1}
(1) 348 K and 300 J (2) 378 K and 300 J
(3) 368 K and 500 J (4) 378 K and 500 J

Q.39 The reaction $A_2 + B_2 \rightarrow 2AB$ follows the mechanism

$A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} A + A$ (fast)

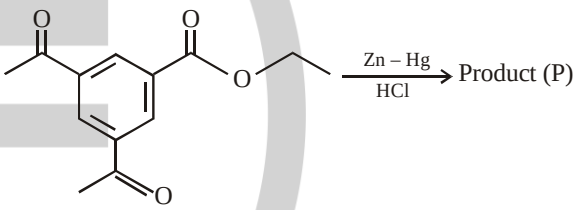
$A + B_2 \xrightarrow{k_2} AB + B$ (slow)

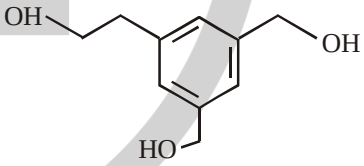
A + B Q AB (fast)
The overall order of the reaction is :
(1) 1.5 (2) 3 (3) 2.5 (4) 2

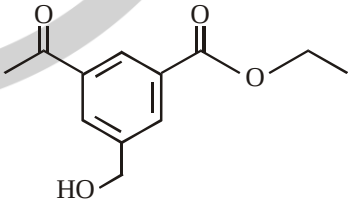
Q.40 If a_0 is denoted as the Bohr radius of hydrogen atom, then what is the de-Broglie wavelength (λ) of the electron present in the second orbit of hydrogen atom? [n : any integer]

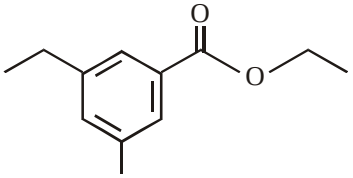
(1) $\frac{2a_0}{n\pi}$ (2) $\frac{8\pi a_0}{n}$ (3) $\frac{4\pi a_0}{n}$ (4) $\frac{4n}{\pi a_0}$

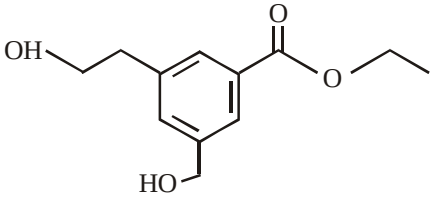
Q.41 The product (P) formed in the following reaction is :



(1) 

(2) 

(3) 

(4) 

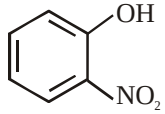
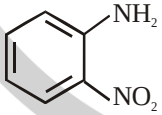
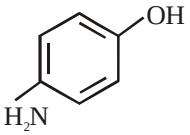
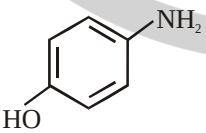
Q.42 An element 'E' has the ionisation enthalpy value of 374 kJ mol^{-1}. 'E' reacts with elements A, B, C and D with electron gain enthalpy values of –328, –349, –325 and –295 kJ mol^{-1}, respectively.
The correct order of the products EA, EB, EC and ED in terms of ionic character is :
(1) EB > EA > EC > ED (2) ED > EC > EA > EB
(3) EA > EB > EC > ED (4) ED > EC > EB > EA

Q.43 Match List - I with List - II.

List–I (Carbohydrate)		List–II (Linkage Source	
(A)	Amylose	(I)	β–C_1–C_4, plant
(B)	Cellulose	(II)	α–C_1–C_4, animal
(C)	Glycogen	(III)	α–C_1–C_4, α–C_1–C_6, plant
(D)	Amylopectin	(IV)	α–C_1–C_4, plant

Choose the correct answer form the options given below :
(1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)
(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(3) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
(4) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

Q.44 The steam volatile compounds among the following are :

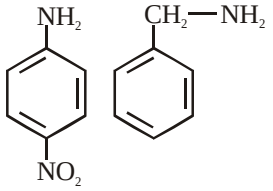
(A) 
(B) 
(C) 
(D) 

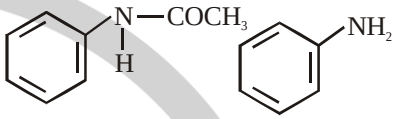
Choose the correct answer from the options given below :
(1) (B) and (D) only (2) (A) and (C) only
(3) (A) and (B) only (4) (A),(B) and (C) only

Q.45 Given below are two statements :
Statement (I): The radii of isoelectronic species increases in the order.
$Mg^{2+} < Na^{+} < F^{-} < O^{2-}$
Statement (II) : The magnitude of electron gain enthalpy of halogen decreases in the order.
Cl > F > Br > I
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Statement I is incorrect but Statement II is correct
(2) Both Statement I and Statement II are incorrect
(3) Statement I is correct but Statement II is incorrect
(4) Both Statement I and Statement II are correct

SECTION-B

Q.46 Given below are some nitrogen containing compounds.



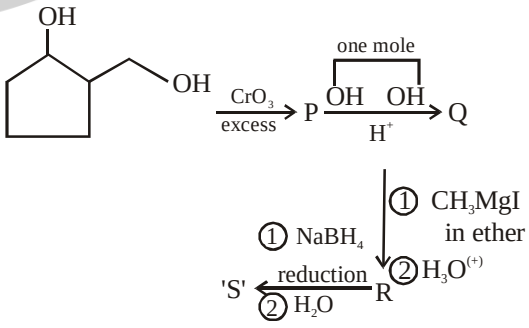


Each of them is treated with HCl separately. 1.0 g of the most basic compound will consume _____ mg of HCl.
(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16, Cl : 35.5)

Q.47 The molar mass of the water insoluble product formed from the fusion of chromite ore ($FeCr_2O_4$) with Na_2CO_3 in presence of O_2 is _____ g mol^{-1}.

Q.48 The sum of sigma (σ) and pi(π) bonds in Hex-1,3-dien-5-yne is _____.

Q.49 If A_2B is 30% ionised in an aqueous solution, then the value of van't Hoff factor (i) is _____ $\times 10^{-1}$.

Q.50 

0.1 mole of compound 'S' will weigh _____g.
(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16)

MATHEMATICS

SECTION-A

Q.51 Let the line x + y = 1 meet the circle $x^2 + y^2 = 4$ at the points A and B. If the line perpendicular to AB and passing through the mid point of the chord AB intersects is the circle at C and D, then the area of the quadrilateral ADBC is equal to

(1) $3\sqrt{7}$ (2) $2\sqrt{14}$ (3) $5\sqrt{7}$ (4) $\sqrt{14}$

Q.52 Let M and m respectively be the maximum and the minimum values of

$$f(x)=\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ \sin^2 x & 1+\cos^2 x & 4\sin 4x \\ \sin^2 x & \cos^2 x & 1+4\sin 4x \end{vmatrix}, x \in R$$

Then $M^4 - m^4$ is equal to :

(1) 1280 (2) 1295 (3) 1040 (4) 1215

Q.53 Two parabolas have the same focus (4, 3) and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersects at the points A and B, then $(AB)^2$ is equal to

(1) 192 (2) 384 (3) 96 (4) 392

Q.54 Let ABC be a triangle formed by the lines 7x – 6y + 3 = 0, x + 2y – 31 = 0 and 9x – 2y – 19 = 0, Let the point (h, k) be the image of the centroid of ΔABC in the line 3x + 6y – 53 = 0. Then $h^2 + k^2 + hk$ is equal to

(1) 37 (2) 47 (3) 40 (4) 36

Q.55 Let $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$ and $(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 168$. Then the maximum value of $|\vec{c}|^2$ is :

(1) 77 (2) 462 (3) 308 (4) 154

Q.56 Let P be the set of seven digit numbers with sum of their digits equal to 11. If the numbers in P are formed by using the digits 1, 2 and 3 only, then the number of elements in the set P is :

(1) 158 (2) 173 (3) 164 (4) 161

Q.57 Let the area of the region $\{(x, y) : 2y \le x^2 + 3, y + |x| \le 3, y \ge |x - 1|\}$ be A. Then 6 A is equal to:

(1) 16 (2) 12 (3) 18 (4) 14

Q.58 The least value of n for which the number of integral terms in the Binomial expansion of $(\sqrt[3]{7} + \sqrt[12]{11})^n$ is 183, is :

(1) 2184 (2) 2148 (3) 2172 (4) 2196

Q.59 The number of solutions of the equation $\left(\frac{9}{x} - \frac{9}{\sqrt{x}} + 2\right)\left(\frac{2}{x} - \frac{7}{\sqrt{x}} + 3\right) = 0$ is :

(1) 2 (2) 4 (3) 1 (4) 3

Q.60 Let y = y(x) be the solution of the differential equation $\cos x(\log_e(\cos x))^2 dy + (\sin x - 3y \sin x \log_e(\cos x))dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{4}\right) = \frac{-1}{\log_e 2}$, then $y\left(\frac{\pi}{6}\right)$ is :

(1) $\frac{2}{\log_e(3) - \log_e(4)}$ (2) $\frac{1}{\log_e(4) - \log_e(3)}$

(3) $-\frac{1}{\log_e(4)}$ (4) $\frac{1}{\log_e(3) - \log_e(4)}$

Q.61 Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by x R y if and only if $\sec^2 x - \tan^2 y = 1$. Then R is :

(1) an equivalence relation

(2) both reflexive and transitive but not symmetric

(3) both reflexive and symmetric but not transitive

(4) reflexive but neither symmetric not transitive

Q.62 Let the ellipse, $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1, A < B$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A, B, C and D, then the area of the quadrilateral ABCD equals:

(1) $6\sqrt{6}$ (2) $\frac{18\sqrt{6}}{5}$ (3) $\frac{12\sqrt{6}}{5}$ (4) $\frac{24\sqrt{6}}{5}$

Q.63 Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11^{th} term is :

(1) 84 (2) 122 (3) 90 (4) 108

Q.64 Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 7\hat{j} + 3\hat{k}$. Let $L_1 : \vec{r} = (-\hat{i} + 2\hat{j} + \hat{k}) + \lambda\vec{a}, \lambda \in R$ and $L_2 : \vec{r} = (\hat{j} + \hat{k}) + \mu\vec{b}, \mu \in R$ be two lines. If the line L_3 passes through the point of intersection of L_1 and L_2, and is parallel to $\vec{a} + \vec{b}$, then L_3 passes through the point:

(1) (8, 26, 12) (2) (2,8, 5)

(3) (–1, –1, 1) (4) (5, 17, 4)

Q.65 The value of $\lim\limits_{n\to\infty}\left(\sum\limits_{K=1}^{n}\frac{k^3+6k^2+11k+5}{(k+3)!}\right)$ is :

(1) $\frac{4}{3}$ (2) 2 (3) $\frac{7}{3}$ (4) $\frac{5}{3}$

Q.66 The integral $80\int\limits_0^{\frac{\pi}{4}}\left(\frac{\sin\theta+\cos\theta}{9+16\sin2\theta}\right)d\theta$ is equal to :

(1) $3\log_e 4$ (2) $6\log_e 4$ (3) $4\log_e 3$ (4) $2\log_e 3$

Q.67 Let $L_1:\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-1}{2}$ and

$L_2:\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z}{1}$ be two lines.

Let L_3 be a line passing through the point (α, β, γ) and be perpendicular to both L_1 and L_2. If L_3 intersects L_1, then $|5\alpha - 11\beta - 8\gamma|$ equals :

(1) 18 (2) 16 (3) 25 (4) 20

Q.68 Let $x_1, x_2, \ldots. x_{10}$ be ten observations such that $\sum\limits_{i=1}^{10}(x_i-2)=30, \sum\limits_{i=1}^{10}(x_i-\beta)^2=98, \beta>2$ and their variance is $\frac{4}{5}$. If μ and σ^2 are respectively the mean and the variance of $2(x_1 - 1) + 4\beta$, $2(x_2 - 1) + 4\beta$, ….. $2(x_{10} - 1) + 4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to :

(1) 100 (2) 110 (3) 120 (4) 90

Q.69 Let $|z_2 - 8 - 2i| \le 1$ and $|z_2 - 2 + 6i| \le 2, z_1, z_2 \in C$. Then the minimum value of $|z_1 - z_2|$ is :

(1) 3 (2) 7 (3) 13 (4) 10

Q.70 Let $A=\left[a_{ij}\right]=\begin{bmatrix}\log_5 128 & \log_4 5\\ \log_5 8 & \log_4 25\end{bmatrix}$.

If A_{ij} is the cofactor of a_{ij}, $C_{ij}=\sum\limits_{k=1}^{2}a_{ik}A_{jk}$, $1\le i, j\le 2$,

and $C = [C_{ij}]$, then $8|C|$ is equal to :

(1) 262 (2) 288 (3) 242 (4) 222

SECTION-B

Q.71 Let $f:(0,\infty)\to R$ be a twice differentiable function. If for some $a\neq 0, \int_0^1 f(\lambda x)d\lambda = af(x)$, f(1) = 1 and $f(16)=\frac{1}{8}$, then $16-f'\left(\frac{1}{16}\right)$ is equal to _____,

Q.72 Let $S=\left\{m\in Z: A^{m^2}+A^m=3I-A^{-6}\right\}$, where $A=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}$. Then n(S) is equal to _____.

Q.73 Let [t] be the greatest integer less than or equal to t. Then the least value of $p \in N$ for which

$$\lim_{x\to 0^+}\begin{pmatrix} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots.+\left[\frac{p}{x}\right]\right)\\ -x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots.+\left[\frac{9^2}{x^2}\right]\right)\end{pmatrix}\ge 1$$

is equal to _____.

Q.74 The number of 6 – letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice, is _____.

Q.75 Let $S=\left\{x:\cos^{-1}x=\pi+\sin^{-1}x+\sin^{-1}(2x+1)\right\}$.

Then $\sum\limits_{x\in S}(2x-1)^2$ is equal to _____.