

JEE-MAIN EXAMINATION-JANUARY 2025

Date: 23/01/2025

Shift : Evening

 PHYSICS
SECTION-A

- Q.1** A ball having kinetic energy KE, is projected at an angle of 60° from the horizontal. What will be the kinetic energy of ball at the highest point of its flight?

(1) $\frac{(KE)}{8}$ (2) $\frac{(KE)}{4}$ (3) $\frac{(KE)}{16}$ (4) $\frac{(KE)}{2}$

- Q.2** Two charges $7\ \mu\text{C}$ and $-4\ \mu\text{C}$ are placed at $(-7\ \text{cm}, 0, 0)$ and $(7\ \text{cm}, 0, 0)$ respectively. Given, $\epsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$, the electrostatic potential energy of the charge configuration is :
(1) $-1.5\ \text{J}$ (2) $-2.0\ \text{J}$ (3) $-1.2\ \text{J}$ (4) $-1.8\ \text{J}$

- Q.3** The refractive index of the material of a glass prism is $\sqrt{3}$. The angle of minimum deviation is equal to the angle of the prism. What is the angle of the prism?
(1) 50° (2) 60° (3) 58° (4) 48°

- Q.4** The equation of a transverse wave travelling along a string is $y(x, t) = 4.0 \sin[20 \times 10^{-3}x + 600t]\ \text{mm}$, where x is in the mm and t is in second. The velocity of the wave is :
(1) $+30\ \text{m/s}$ (2) $-60\ \text{m/s}$ (3) $-30\ \text{m/s}$ (4) $+60\ \text{m/s}$

- Q.5** The energy of a system is given as $E(t) = \alpha^3 e^{-\beta t}$, where t is the time and $\beta = 0.3\ \text{s}^{-1}$. The errors in the measurement of α and t are 1.2% and 1.6%, respectively. At $t = 5\ \text{s}$, maximum percentage error in the energy is :
(1) 4% (2) 11.6% (3) 6% (4) 8.4%

- Q.6** In photoelectric effect an em-wave is incident on a metal surface and electrons are ejected from the surface. If the work function of the metal is 2.14 eV and stopping potential is 2 V, what is the wavelength of the em-wave? (Given $hc = 1242\ \text{eVnm}$ where h is the Planck's constant and c is the speed of light in vacuum.)
(1) 400 nm (2) 600 nm (3) 200 nm (4) 300 nm

- Q.7** A circular disk of radius R meter and mass M kg is rotating around the axis perpendicular to the disk. An external torque is applied to the disk such that $\theta(t) = 5t^2 - 8t$, where $\theta(t)$ is the angular position of the rotating disc as a function of time t .

How much power is delivered by the applied torque, when $t = 2\ \text{s}$?

(1) $60\ \text{MR}^2$ (2) $72\ \text{MR}^2$ (3) $108\ \text{MR}^2$ (4) $8\ \text{MR}^2$

- Q.8** Water flows in a horizontal pipe whose one end is closed with a valve. The reading of the pressure gauge attached to the pipe is P_1 . The reading of the pressure gauge falls to P_2 when the valve is opened. The speed of water flowing in the pipe is proportional to

(1) $\sqrt{P_1 - P_2}$ (2) $(P_1 - P_2)^2$
(3) $(P_1 - P_2)^4$ (4) $P_1 - P_2$

- Q.9** Match List-I with List-II.

List-I	List-II
(A) Permeability of free space	(I) $[\text{M L}^2 \text{T}^{-2}]$
(B) Magnetic field	(II) $[\text{M L}^{-2} \text{A}^{-1}]$
(C) Magnetic moment	(III) $[\text{M L T}^{-2} \text{A}^{-2}]$
(D) Torsional constant	(IV) $[\text{L}^2 \text{A}]$

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)
(2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

- Q.10** If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what is the time period of rotation of the satellite? Given rotational time period of Moon = 27 days and gravitational attraction between the satellite and the moon is neglected.

(1) 1 day (2) 81 days (3) 27 days (4) 3 days

- Q.11** Two point charges $-4\ \mu\text{C}$ and $4\ \mu\text{C}$, constituting an electric dipole, are placed at $(-9, 0, 0)\ \text{cm}$ and $(9, 0, 0)\ \text{cm}$ in a uniform electric field of strength $10^4\ \text{NC}^{-1}$. The work done on the dipole in rotating it from the equilibrium through 180° is :

(1) 14.4 mJ (2) 18.4 mJ (3) 12.4 mJ (4) 16.4 mJ

- Q.12** A galvanometer having a coil of resistance $30\ \Omega$ need 20 mA of current for full-scale deflection. If a maximum current of 3 A is to be measured using this galvanometer, the resistance of the shunt to be added

to the galvanometer should be $\frac{30}{X}\ \Omega$, where X is

(1) 447 (2) 298 (3) 149 (4) 596

- Q.13** The width of one of the two slits in Young's double slit experiment is d while that of the other slit is xd . If the ratio of the maximum to the minimum intensity in the interference pattern on the screen is $9 : 4$ then what is the value of x ?

(Assume that the field strength varies according to the slit width.)

- (1) 2 (2) 3 (3) 5 (4) 4

- Q.14** Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : The binding energy per nucleon is found to be nearly independent of the atomic number A , for nuclei with mass numbers between 30 and 170.

Reason (R) : Nuclear force is long range.

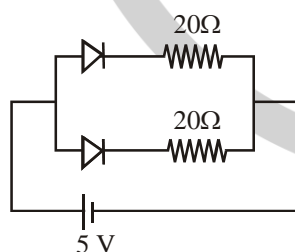
In the light of the above statements, choose the correct answer from the options given below :

- (1) (A) is false but (R) is true
(2) (A) is true but (R) is false
(3) Both (A) and (R) are true and (R) is the correct explanation of (A)
(4) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

- Q.15** Water of mass m gram is slowly heated to increase the temperature from T_1 to T_2 . The change in entropy of the water, given specific heat of water is $1 \text{ J kg}^{-1} \text{ K}^{-1}$, is :

- (1) zero (2) $m(T_2 - T_1)$
(3) $m \ln \left(\frac{T_1}{T_2} \right)$ (4) $m \ln \left(\frac{T_2}{T_1} \right)$

- Q.16** What is the current through the battery in the circuit shown below?

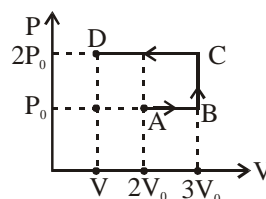


- (1) 1.0 A (2) 1.5 A
(3) 0.5 A (4) 0.25 A

- Q.17** A plane electromagnetic wave of frequency 20 MHz travels in free space along the $+x$ direction. At a particular point in space and time, the electric field vector of the wave is $E_y = 9.3 \text{ Vm}^{-1}$. Then, the magnetic field vector of the wave at that point is-

- (1) $B_z = 9.3 \times 10^{-8} \text{ T}$
(2) $B_z = 1.55 \times 10^{-8} \text{ T}$
(3) $B_z = 6.2 \times 10^{-8} \text{ T}$
(4) $B_z = 3.1 \times 10^{-8} \text{ T}$

Q.18



Using the given P-V diagram, the work done by an ideal gas along the path ABCD is-

- (1) $4P_0V_0$ (2) $3P_0V_0$ (3) $-4P_0V_0$ (4) $-3P_0V_0$

Q.19

A concave mirror of focal length f in air is dipped in a liquid of refractive index μ . Its focal length in the liquid will be :

- (1) $\frac{f}{\mu}$ (2) $\frac{f}{(\mu-1)}$ (3) μf (4) f

Q.20

A massless spring gets elongated by amount x_1 under a tension of 5 N. Its elongation is x_2 under the tension of 7 N. For the elongation of $(5x_1 - 2x_2)$, the tension in the spring will be,

- (1) 15 N (2) 20 N (3) 11 N (4) 39 N

SECTION-B

Q.21

An air bubble of radius 1.0 mm is observed at a depth of 20 cm below the free surface of a liquid having surface tension 0.095 J/m^2 and density 103 kg/m^3 . The difference between pressure inside the bubble and atmospheric pressure _____ N/m^2 . (Take $g = 10 \text{ m/s}^2$)

Q.22

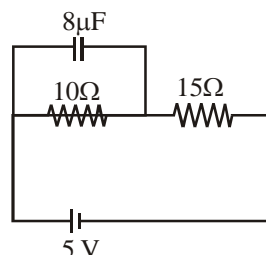
A satellite of mass $\frac{M}{2}$ is revolving around earth in a circular orbit at a height of $\frac{R}{3}$ from earth surface. The

angular momentum of the satellite is $M\sqrt{\frac{GMR}{x}}$. The

value of x is _____, where M and R are the mass and radius of earth, respectively. (G is the gravitational constant)

Q.23

At steady state the charge on the capacitor, as shown in the circuit below, is _____ μC .



Q.24 A time varying potential difference is applied between the plates of a parallel plate capacitor of capacitance $2.5 \mu\text{F}$. The dielectric constant of the medium between the capacitor plates is 1. It produces an instantaneous displacement current of 0.25 mA in the intervening space between the capacitor plates, the magnitude of the rate of change of the potential difference will be _____ Vs^{-1} .

Q.25 In a series LCR circuit, a resistor of 300Ω , a capacitor of 25 nF and an inductor of 100 mH are used. For maximum current in the circuit, the angular frequency of the ac source is _____ $\times 10^4 \text{ radians s}^{-1}$.

CHEMISTRY SECTION-A

Q.26 The effect of temperature on spontaneity of reactions are represented as:

	ΔH	ΔS	Temperature	Spontaneity
(A)	+	-	any T	Non spontaneous
(B)	+	+	low T	spontaneous
(C)	-	-	low T	Non spontaneous
(D)	-	+	any T	spontaneous

Select the incorrect option about spontaneity

- (1) (B) and (D) only (2) (A) and (D) only
(3) (B) and (C) only (D) (A) and (C) only

Q.27 Standard electrode potentials for a few half cells are mentioned below:

$$E_{\text{Cu}^{2+}/\text{Cu}}^{\circ} = 0.34 \text{ V}, E_{\text{Zn}^{2+}/\text{Zn}}^{\circ} = -0.76 \text{ V}$$

$$E_{\text{Ag}^{+}/\text{Ag}}^{\circ} = 0.80 \text{ V}, E_{\text{Mg}^{2+}/\text{Mg}}^{\circ} = -2.37 \text{ V}$$

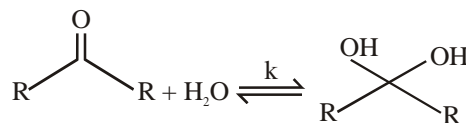
Which one of the following cells gives the most negative value of ΔG° ?

- (1) $\text{Zn} | \text{Zn}^{2+} (1\text{M}) || \text{Ag}^{+} (1\text{M}) | \text{Ag}$
(2) $\text{Zn} | \text{Zn}^{2+} (1\text{M}) || \text{Mg}^{2+} (1\text{M}) | \text{Mg}$
(3) $\text{Ag} | \text{Ag}^{+} (1\text{M}) || \text{Mg}^{2+} (1\text{M}) | \text{Mg}$
(4) $\text{Cu} | \text{Cu}^{2+} (1\text{M}) || \text{Ag}^{+} (1\text{M}) | \text{Ag}$

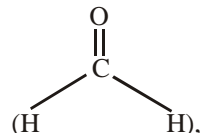
Q.28 The α - Helix and β - Pleated sheet structures of protein are associated with its:

- (1) quaternary structure
(2) primary structure
(3) secondary structure
(4) tertiary structure

Q.29 Given below are two statements:
Consider the following reaction

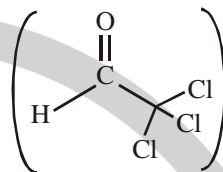


Statement (I): In the case of formaldehyde



K is about 2280, due to small substituents, hydration is faster.

Statement (II) : In the case of trichloro acetaldehyde

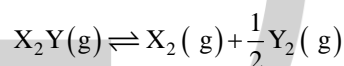


of -Cl.

In the light of the above statements, choose the correct answer from the options given below:

- (1) Statement I true but Statement II is false
(2) Both Statement I and Statement II are true
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are false

Q.30 Consider the reaction

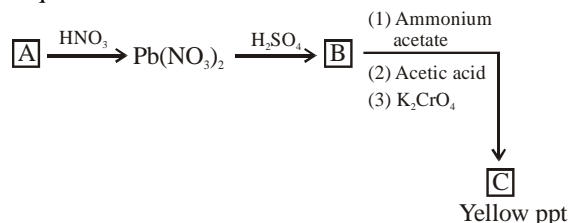


The equation representing correct relationship between the degree of dissociation (x) of $\text{X}_2\text{Y}(\text{g})$ with its equilibrium constant K_p is _____.

Assume x to be very very small.

- (1) $x = \sqrt[3]{\frac{2K_p}{p}}$ (2) $x = \sqrt[3]{\frac{2K_p^2}{p}}$
(3) $x = \sqrt[3]{\frac{K_p}{2p}}$ (4) $x = \sqrt[3]{\frac{K_p}{p}}$

Q.31 Identify A, B and C in the given below reaction sequence



- (1) $\text{PbCl}_2, \text{PbSO}_4, \text{PbCrO}_4$
(2) $\text{PbS}, \text{PbSO}_4, \text{PbCrO}_4$
(3) $\text{PbS}, \text{PbSO}_4, \text{Pb}(\text{CH}_3\text{COO})_2$
(4) $\text{PbCl}_2, \text{Pb}(\text{SO}_4)_2, \text{PbCrO}_4$

Q.32 Given below are two statements:
Statement (I): The boiling points of alcohols and phenols increase with increase in the number of C - atoms. Statement (II): The boiling points of alcohols and phenols are higher in comparison to other class of compounds such as ethers, haloalkanes.

In the light of the above statements, choose the correct answer from the options given below:

- (1) Both Statement I and Statement II are false
- (2) Statement I is false but Statement II is true
- (3) Statement I is true but Statement II is false
- (4) Both Statement I and Statement II are true

Q.33 When a non-volatile solute is added to the solvent, the vapour pressure of the solvent decreases by 10 mm of Hg. The mole fraction of the solute in the solution is 0.2. What would be the mole fraction of the solvent if decrease in vapour pressure is 20 mm of Hg ?

- (1) 0.6
- (2) 0.4
- (3) 0.2
- (4) 0.8

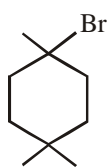
Q.34 Given below are two statements:
Statement (I): For a given shell, the total number of allowed orbitals is given by n^2 .

Statement (II): For any subshell, the spatial orientation of the orbitals is given by $-l$ to $+l$ values including zero.

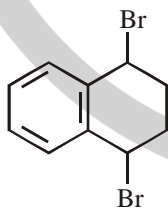
In the light of the above statements, choose the correct answer from the options given below:

- (1) Statement I is true but Statement II is false
- (2) Statement I is false but Statement II is true
- (3) Both Statement I and Statement II are true
- (4) Both Statement I and Statement II are false

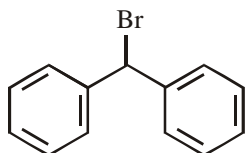
Q.35 The ascending order of relative rate of solvolysis of following compounds is



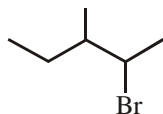
(A)



(B)



(C)



(D)

- (1) (D) < (A) < (B) < (C)
- (2) (C) < (B) < (A) < (D)
- (3) (D) < (B) < (A) < (C)
- (4) (C) < (D) < (B) < (A)

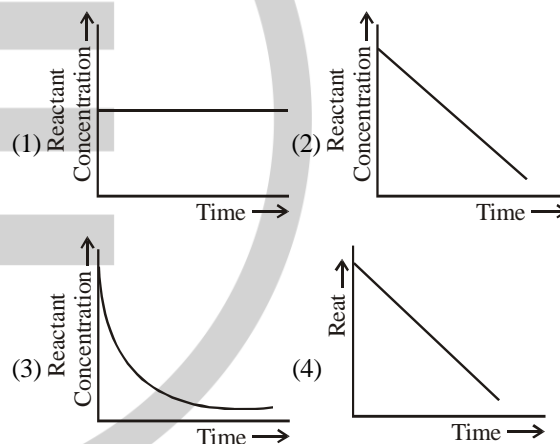
Q.36 Match List - I with List - II

List-I (Isomers of $C_{10}H_{14}$)	List-II (Ozonolysis] product)
(A)	(I)
(B)	(II)
(C)	(III)
(D)	(IV)

Choose the correct answer from the options given below :

- (1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
- (2) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
- (3) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)
- (4) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

Q.37 Which of the following graphs most appropriately represents a zero order reaction?



Q.38 Match List - I with List - II.

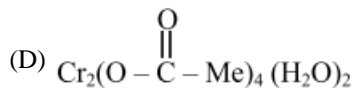
List - I	List - II
(A) Bronze	(I) Cu, Ni
(B) Brass	(II) Fe, Cr, Ni, C
(C) UK silver coin	(III) Cu, Zn
(D) Stainless Steel	(IV) Cu, Sn

Choose the correct answer from the options given below :

- (1) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
- (2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
- (3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
- (4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Q.39 Identify the coordination complexes in which the central metal ion has d^4 configuration.

- (A) $[\text{FeO}_4]^{2-}$
(B) $[\text{Mn}(\text{CN})_6]^{3-}$
(C) $[\text{Fe}(\text{CN})_6]^{3-}$



(E) $[\text{NiF}_6]^{2-}$

Choose the correct answer from the options given below :

- (1) (C) and (E) only (2) (B), (C) and (D) only
(3) (B) and (D) only (4) (A), (B) and (E) only

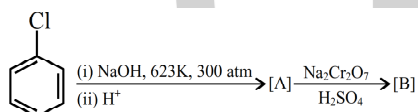
Q.40 Given below are the atomic numbers of some group 14 elements. The atomic number of the element with lowest melting point is :

- (1) 14 (2) 6 (3) 82 (4) 50

Q.41 pH of water is 7 at 25°C . If water is heated to 80°C , it's pH will :

- (1) Decrease
(2) Remains the same
(3) H^+ concentration increases, OH^- concentration decreases
(4) Increase

Q.42 Identify the products [A] and [B], respectively in the following reaction :



- (1) [A] , [B] (2) [A] , [B]
(3) [A] , [B] (4) [A] , [B]

Q.43 Consider a binary solution of two volatile liquid components 1 and 2, x_1 and y_1 are the mole fractions of component 1 in liquid and vapour phase, respectively.

The slope and intercept of the linear plot of $\frac{1}{x_1}$ vs $\frac{1}{y_1}$ are given respectively as :

- (1) $\frac{P_1^0}{P_2^0}, \frac{P_2^0 - P_1^0}{P_2^0}$ (2) $\frac{P_2^0}{P_1^0}, \frac{P_1^0 - P_2^0}{P_2^0}$
(3) $\frac{P_1^0}{P_2^0}, \frac{P_1^0 - P_2^0}{P_2^0}$ (4) $\frac{P_2^0}{P_1^0}, \frac{P_2^0 - P_1^0}{P_2^0}$

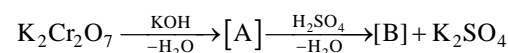
Q.44 Given below are two statements about X-ray spectra of elements :

Statement (I): A plot of $\sqrt{\nu}$ (ν = frequency of X-rays emitted) vs atomic mass is a straight line.

Statement (II) : A plot of ν (ν = frequency of X-rays emitted) vs atomic number is a straight line. In the light of the above statements choose the correct answer from the options given below :

- (1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is false but Statement II is true

Q.45 Consider the following reactions



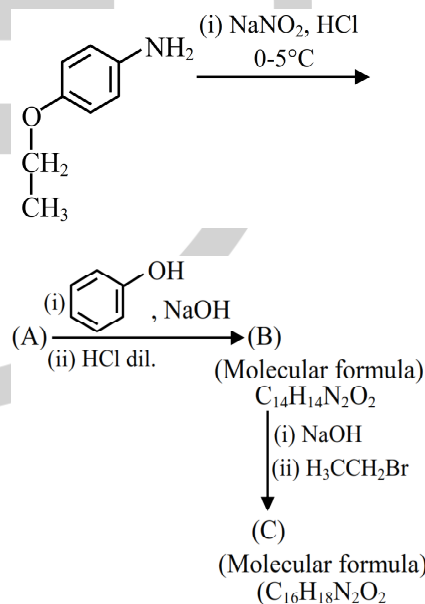
The products [A] and [B], respectively are :

- (1) $\text{K}_2\text{Cr}(\text{OH})_6$ and Cr_2O_3
(2) K_2CrO_4 and Cr_2O_3
(3) K_2CrO_4 and $\text{K}_2\text{Cr}_2\text{O}_7$
(4) K_2CrO_4 and CrO

SECTION-B

Q.46 0.01 mole of an organic compound (X) containing 10% hydrogen, on complete combustion produced 0.9 g H_2O . Molar mass of (X) is ____ g mol^{-1} .

Q.47 Consider the following sequence of reactions.



Total number of sp^3 hybridised carbon atoms in the major product C formed is _____.

Q.48 When 81.0 g of aluminium is allowed to react with 128.0 g of oxygen gas, the mass of aluminium oxide produced in grams is _____. (Nearest integer)

Given :

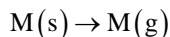
Molar mass of Al is 27.0 g mol^{-1}

Molar mass of O is 16.0 g mol^{-1}

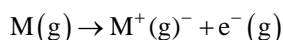
- Q.49** The bond dissociation enthalpy of $X_2\Delta H_{\text{bond}}^0$ calculated from the given data is _____ kJ mol^{-1} . (Nearest integer)



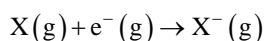
$$\Delta H_{\text{lattice}}^0 = 800 \text{ kJ mol}^{-1}$$



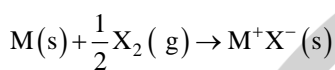
$$\Delta H_{\text{sub}}^0 = 100 \text{ kJ mol}^{-1}$$



$$\Delta H_i^0 = 500 \text{ kJ mol}^{-1}$$



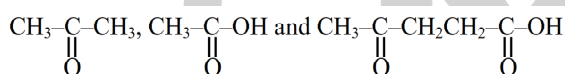
$$\Delta H_{\text{eg}}^0 = -300 \text{ kJ mol}^{-1}$$



$$\Delta H_f^0 = -400 \text{ kJ mol}^{-1}$$

[Given : M^+X^- is a pure ionic compound and X forms a diatomic molecule X_2 is gaseous state]

- Q.50** A compound 'X' absorbs 2 moles of hydrogen and 'X' upon oxidation with KMnO_4/H^+ gives



The total number of σ bonds present in the compound 'X' is _____.

MATHEMATICS SECTION-A

- Q.51** If in the expansion of $(1+x)^p(1-x)^q$, the coefficients of x and x^2 are 1 and -2, respectively, then $p^2 + q^2$ is equal to :
(1) 8 (2) 18 (3) 13 (4) 20

- Q.52** Let $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x+y| \geq 3\}$ and $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x|+|y| \leq 3\}$.

If $C = \{(x, y) \in A \cap B : x = 0 \text{ or } y = 0\}$, then

$$\sum_{(x,y) \in C} |x+y| \text{ is :}$$

- (1) 15 (2) 18 (3) 24 (4) 12

- Q.53** The system of equations $x+y+z=6$, $x+2y+5z=9$, $x+5y+\lambda z=\mu$, has no solution if
(1) $\lambda=17, \mu \neq 18$ (2) $\lambda \neq 17, \mu \neq 18$
(3) $\lambda=15, \mu \neq 17$ (4) $\lambda=17, \mu=18$

- Q.54** Let $\int x^3 \sin x dx = g(x) + C$, where C is the constant of integration.

$$\text{If } 8 \left(g \left(\frac{\pi}{2} \right) + g' \left(\frac{\pi}{2} \right) \right) = \alpha \pi^3 + \beta \pi^2 + \gamma, \alpha, \beta, \gamma \in \mathbb{Z},$$

Then $\alpha + \beta - \gamma$ equals :

- (1) 55 (2) 47 (3) 48 (4) 62

- Q.55** A rod of length eight units moves such that its ends A and B always lie on the lines $x-y+2=0$ and $y+2=0$, respectively. If the locus of the point P, that divides the rod AB internally in the ratio 2 : 1 is $9(x^2 + \alpha y^2 + \beta xy + \gamma x + 28y) - 76 = 0$, then $\alpha - \beta - \gamma$ is equal to :
(1) 24 (2) 23 (3) 21 (4) 22

- Q.56** The distance of the line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ from the point $(1, 4, 0)$ along the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is :
(1) $\sqrt{17}$ (2) $\sqrt{14}$ (3) $\sqrt{15}$ (4) $\sqrt{13}$

- Q.57** Let the point A divide the line segment joining the points $P(-1, -1, 2)$ and $Q(5, 5, 10)$ internally in the ratio $r : 1$ ($r > 0$). If O is the origin and $(\overrightarrow{OQ} \cdot \overrightarrow{OA}) - \frac{1}{5} |\overrightarrow{OP} \times \overrightarrow{OA}|^2 = 10$, then the value of r is :
(1) 14 (2) 3 (3) $\sqrt{7}$ (4) 7

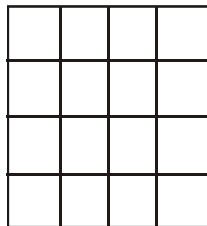
- Q.58** If the area of the region $\{(x, y) : -1 \leq x \leq 1, 0 \leq y \leq a + e^{|x|} - e^{-x}, a > 0\}$ is

$$\frac{e^2 + 8e + 1}{e}, \text{ then the value of } a \text{ is :}$$

- (1) 7 (2) 6 (3) 8 (4) 5

- Q.59** A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm, the ice-cream melts at the rate of $81 \text{ cm}^3/\text{min}$ and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi} \text{ cm/min}$. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is :
(1) 225π (2) 128π (3) 196π (4) 256π

Q.60 A board has 16 squares as shown in the figure :



Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is :

- (1) $\frac{4}{5}$ (2) $\frac{7}{10}$ (3) $\frac{3}{5}$ (4) $\frac{23}{30}$

Q.61 Let $x = x(y)$ be the solution of the differential equation

$$y = \left(x - y \frac{dx}{dy} \right) \sin \left(\frac{x}{y} \right), y > 0 \text{ and } x(1) = \frac{\pi}{2}.$$

Then $\cos(x(2))$ is equal to :

- (1) $1 - 2(\log_e 2)^2$ (2) $2(\log_e 2)^2 - 1$
(3) $2(\log_e 2) - 1$ (4) $1 - 2(\log_e 2)$

Q.62 Let the range of the function

$$f(x) = 6 + 16 \cos x \cdot \cos \left(\frac{\pi}{3} - x \right) \cdot \cos \left(\frac{\pi}{3} + x \right).$$

$\sin 3x \cdot \cos 6x, x \in \mathbb{R}$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is :

- (1) 11 (2) 8 (3) 10 (4) 9

Q.63 Let the shortest distance from $(a, 0), a > 0$, to the parabola $y^2 = 4x$ be 4. Then the equation of the circle passing through the point $(a, 0)$ and the focus of the parabola, and having its centre on the axis of the parabola is:

- (1) $x^2 + y^2 - 6x + 5 = 0$ (2) $x^2 + y^2 - 4x + 3 = 0$
(3) $x^2 + y^2 - 10x + 9 = 0$ (4) $x^2 + y^2 - 8x + 7 = 0$

Q.64 Let $X = \mathbb{R} \times \mathbb{R}$. Define a relation R on X as:

$$(a_1, b_1) R (a_2, b_2) \Leftrightarrow b_1 = b_2.$$

Statement-I: R is an equivalence relation.

Statement-II: For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y) R (a, b)\}$ represents a line parallel to $y = x$.

In the light of the above statements, choose the correct answer from the options given below:

- (1) Both Statement-I and Statement-II are false.
(2) Statement-I is true but Statement-II is false.
(3) Both Statement-I and Statement-II are true.
(4) Statement-I is false but Statement-II is true.

Q.65 The length of the chord of the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$, whose mid-point is $\left(1, \frac{1}{2}\right)$, is:

- (1) $\frac{2}{3}\sqrt{15}$ (2) $\frac{5}{3}\sqrt{15}$ (3) $\frac{1}{3}\sqrt{15}$ (4) $\sqrt{15}$

Q.66 Let $A = [a_{ij}]$ be a 3×3 matrix such that

$$A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, A \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } A \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \text{ then } a_{23}$$

equals:

- (1) -1 (2) 0 (3) 2 (4) 1

Q.67 The number of complex numbers z , satisfying $|z| = 1$

$$\text{and } \left| \frac{z}{\bar{z}} + \frac{\bar{z}}{z} \right| = 1, \text{ is :}$$

- (1) 6 (2) 4 (3) 10 (4) 8

Q.68 If the square of the shortest distance between the lines

$$\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-3} \text{ and } \frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5} \text{ is } \frac{m}{n},$$

where m, n are coprime numbers, then $m + n$ is equal to:

- (1) 6 (2) 9 (3) 21 (4) 14

Q.69 If $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$,

$$\text{then } \int_0^{21} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx \text{ equals:}$$

- (1) $\frac{\pi^2}{16}$ (2) $\frac{\pi^2}{4}$ (3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{12}$

Q.70 $\lim_{x \rightarrow \infty} \frac{(2x^2 - 3x + 5)(3x - 1)^{\frac{x}{2}}}{(3x^2 + 5x + 4)\sqrt{(3x + 2)^x}}$ is equal to:

- (1) $\frac{2}{\sqrt{3e}}$ (2) $\frac{2e}{\sqrt{3}}$ (3) $\frac{2e}{3}$ (4) $\frac{2}{3\sqrt{e}}$

SECTION-B

Q.71 The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is _____.

Q.72 Let α, β be the roots of the equation $x^2 - ax - b = 0$ with $\text{Im}(\alpha) < \text{Im}(\beta)$. Let $P_n = \alpha^n - \beta^n$. If $P_3 = -5\sqrt{7}i$, $P_4 = -3\sqrt{7}i$, $P_5 = 11\sqrt{7}i$ and $P_6 = 45\sqrt{7}i$, then $|\alpha^4 + \beta^4|$ is equal to .

Q.73 The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5. If the values of λ , for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and λ_2 , $\lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to _____.

Q.74 The variance of the numbers 8, 21, 34, 47,, 320, is _____.

Q.75 The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2p$ is equal to _____.

